

**ORIGINAL RESEARCH PAPER**

# Investigation of the Effect of Nanoparticles Mean Diameter on Turbulent Mixed Convection of a Nanofluid in a Horizontal Curved tube Using a Two Phase Approach

O. Ghaffari<sup>1</sup>, A. Behzadmehr<sup>1,\*</sup>

<sup>1</sup>Mechanical Engineering Department, University of Sistan and Baluchestan, Zahedan, I.R. Iran

## Abstract

Turbulent mixed convection of a nanofluid (water/Al<sub>2</sub>O<sub>3</sub>,  $\Phi=0.02$ ) has been studied numerically. Two-phase mixture model has been used to investigate the effects of nanoparticles mean diameter on the flow parameters. Nanoparticles distribution at the tube cross section shows that the particles are uniformly dispersed. The non-uniformity of the particles distribution occurs in the case of large nanoparticles and/or high value of the Grashof numbers. The study of particle size effect showed that the effective Nusselt number and turbulent intensity increases with the decreased of particle size.

**Keywords:** Nanofluid; Turbulent Mixed Convection; Two phase; Curved tube; nanoparticles mean diameter; pressure

drop

## 1. Introduction

The efficiency and compactness of various thermal management equipments are seriously limited by the low thermal conductivity of conventional heat transfer fluids such as water, oil, and ethylene glycol mixture. Hence, several researches have been conducted to develop advanced heat transfer fluids with substantially higher thermal characteristic. Choi [1] presented, a new generation of solid-liquid mixtures that is called nanofluid. Nanofluid is a uniformly suspending nanoparticles with average sizes below 100 nm in a base fluid that shows a significant improvement in the thermal properties of the base fluid. Over the years, researchers have extensively studied the thermal conductivity of nanofluids. They found an

anomalous increase in the thermal conductivity even at low volume concentrations of suspended nanoparticles [ 2-4 ]. Many attempts have been made to formulate theoretical models for the prediction of the effective thermal conductivity [5-7]. In addition to determine the effective thermal conductivity, the conductive heat transfer coefficient must be well known to be able using in thermal devices. Convective heat transfer with nanofluids can be modeled using the two-phase or single phase approach. The first provides the possibility of understanding the functions of both the fluid phase and the solid particles in the heat transfer process. The second assumes that the fluid phase and particles are in thermal equilibrium and moves with the same velocity. This approach is simpler and requires less computational time. Thus it has been used in several theoretical studies of convective heat transfer with nanofluids [8-10]. However, the concerns in single phase modeling is selecting the proper effective

\*Corresponding author  
Email Address: amin.behzadmehr@eng.usb.ac.ir

Nomenclature		Greek letters	
a	radius of curved tube (m)	$\alpha$	Thermal diffusivity
$C_{f_m}$	peripheral average friction Coefficient	$\beta$	Volumetric expansion coefficient ( $k^{-1}$ )
$C_p$	Specific heat capacity	$\delta$	Curvature ratio ( $= a/R_c$ )
$d_p$	nanoparticle diameter	$\varepsilon$	Dissipation of turbulent kinetic energy
D	diameter of curved tube (m)	$\theta$	Volume fraction
De	Dean number ( $Re(\frac{a}{R_c})^5$ )	$\lambda$	Thermal conductivity ( $Wm^{-1}k^{-1}$ )
g	Acceleration of gravity ( $m s^{-2}$ )	$\mu$	Dynamic viscosity ( $Nm s^{-2}$ )
Gr	Grashof number	<b>Subscripts</b>	
I	Turbulent intensity	b	bulk
k	Turbulent kinetic energy	eff	effective
$Nu_m$	Peripheral average Nusselt number on a cross section	f	primary phase
$Nu_\theta$	Local Nusselt number on the Circumference of a pipe. [ $= (\frac{q_w D}{\lambda_{eff}}) / (T_w - T_b)$ ]	fd	fully developed
P	Pressure	k	the kth phase
$R_c$	Curvature radius of tube	m	mean
Re	Reynolds number	p	particle, secondary phase
T,t	time-mean and fluctuating velocity ( $m s^{-1}$ )	s	solid
$x_i$	Cartesian coordinate in i direction	t	turbulent
Z	Axial coordinate (m)	w	wall
		z	axial direction
		$\theta$	tangential direction
		0	inlet condition

properties for nanofluids and taking into account the chaotic movement of ultra fine particles. To partially overcome this difficulty, some researches [11-13] have used the dispersion model which takes into account the improvement of heat transfer due to the random movement of particles in the main flow. In addition several factors such as gravity, friction between the fluid and solid particles, Brownian forces, the phenomena of Brownian diffusion, sedimentation, and dispersion may coexist in the main flow of a nanofluid. This means that the slip velocity between the fluid and particles may not be zero [14]. Buongiorno [15] has purposed seven slip mechanisms in order to develop a realistic two-component model for transport phenomena in nanofluid. He refuted the assumption of dispersion

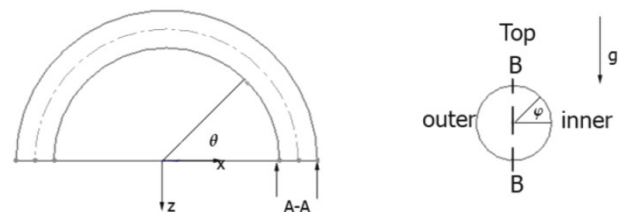
model to simulate nanofluid and argued that heat transfer enhancement from nanoparticle dispersion is completely negligible in nanofluids. Furthermore, he showed that the particles move homogeneously with the fluid in the presence of turbulent eddies. Therefore it seems that the two-phase approach could better model nanofluid behaviors. Behzadmehr et al. [16] studied the turbulent forced convection of a nanofluid in a circular tube by using a two-phase approach. They implemented the two-phase mixture model for the first time to study nanofluid. Their comparison with the experimental results showed that the two-phase mixture model is more precise than the single phase model. Lotfi et al. [17] studied two-phase Eulerian model to investigate such a flow field. The comparison of their numerical results with

experimental values shows that the mixture model is more precise than the two phase Eulerian model. Bianco et al. [18 ] confirmed that the two phase mixture model is suitable for the simulation of nanofluid. Kondaraju et al.[19 ] used a combined Eulerian and Lagrangian method to investigate heat transfer in turbulent nanofluids. They reported that the influence of particle dispersion on convective heat transfer of nanofluids is not significant. They showed that the two-way interaction between the fluid and particle temperature is the main reason for the Nusselt number enhancement. The particles used in the simulations were fairly large. Further study using smaller particle diameters is necessary to analyze the effect of particle dispersion on the convective heat transfer of nanofluids. Mirmasomi and Behzadmehr [20] investigated the effect of nanoparticles mean diameter on laminar mixed convection heat transfer of a nanofluid in a horizontal tube. Akbarinia and Lure [21] investigated the effect of particle size on the flow behavior of a nanofluid in a curved tube. The potential applications of curved geometries in process industry reviewed by Subhashini et al.[22].The role of curved tubes is expected to gain significant importance in near future due to their advantages such enhanced cross-sectional mixing, reduction in axial dispersion, higher heat and mass transfer. Berger et al [23] and Shah and Joshi [24] reviewed many experimental, theoretical and numerical studies on turbulent heat transfer in curved pipes; most of them were limited to fully developed flow. Hogg [25] theoretically calculated the velocity and temperature profile for air flow. Their results showed good agreement with the numerical prediction conducted by Patankar et al. [26]. Li et al. [27] studied turbulent mixed convection heat transfer in the entrance region of a curved pipe with uniform wall temperature. Ghaffari et al [28] used two phase mixture model to study the turbulent mixed convection flow of a nanofluid in the entrance region of a curved tube. They investigated the effect of volume fraction on the thermal and hydrodynamic parameters of a nanofluid. They reported that in higher Gr, where the effect of buoyancy induced secondary flow becomes important, using higher nanoparticle concentration decreases the flow turbulent intensity across the vertical plane. Following that work, in this paper two phase mixture model is used to study the effect of nanoparticle size on turbulent mixed convection flow of a nanofluid in the entrance region of a curved tube

with constant wall-temperature at different Gr number. The effects of nanoparticle mean diameter on the heat transfer augmentation are studied. The secondary flow, turbulent intensity, wall shear stress and pressure drop are presented.

## 2. Mathematical modeling

Turbulent Mixed convection of a nanofluid consisting of water and  $Al_2O_3$  in a horizontal curved tube with constant wall- temperature at the solid–liquid interface has been considered. Fig. 1 shows the geometry of the considered problem. The computation domain is composed of a 180 degree curved circular pipe with radius  $a$  and sectional angle  $\varphi$ . Gravitational force is exerted in the vertical direction. The physical properties of the fluid are assumed to be constant except for the density in the momentum equation, which varies linearly with the temperature (Boussinesq’s hypothesis). The mixture model, based on a single fluid two phase approach, is employed in the simulation by assuming that the coupling between phases is strong, and particles closely follow the flow. The two phases are assumed to be interpenetrating meaning that each phase has its own velocity vector field, and within any control volume there is a volume fraction of primary phase and also a volume fraction of the secondary phase. Instead of utilizing the governing equations of each separately, the continuity, momentum and energy equations for the mixture are employed.



**Fig.1.** Schematic of the considered horizontal curved tube,  $\delta = .05$

The dimensional equations for steady state mean conditions are

$$\nabla \cdot (\rho_m \mathbf{V}_m) = 0 \quad (1)$$

$$\begin{aligned} \nabla \cdot (\rho_m \mathbf{V}_m \mathbf{V}_m) &= -\nabla p_m + \nabla \cdot [\boldsymbol{\tau} - \boldsymbol{\tau}_t] \\ -\rho_{\text{eff}} \beta_{\text{eff}} (T - T_o) \mathbf{g} & \\ + \nabla \cdot \left( \sum_{k=1}^n \Phi_k \rho_k \mathbf{V}_{\text{dr},k} \mathbf{V}_{\text{dr},k} \right) & \end{aligned} \quad (2)$$

$$\nabla \cdot (\Phi_k \mathbf{V}_k (\rho_k h_k + p)) = \nabla \cdot (\lambda_{\text{eff}} \nabla T - C_p \rho_m \overline{v_t}) \quad (3)$$

Volume fraction:

$$\nabla \cdot (\Phi_p \rho_p \mathbf{V}_m) = -\nabla \cdot (\Phi_p \rho_p \mathbf{V}_{\text{dr},p}) \quad (4)$$

where

$$\begin{aligned} \mathbf{V}_m &= \frac{\sum_{k=1}^n \Phi_k \rho_k \mathbf{V}_k}{\rho_{\text{eff}}} \quad \rho_m = \sum_{k=1}^n \Phi_k \mu_k \\ \mu_m &= \sum_{k=1}^n \Phi_k \mu_k \end{aligned} \quad (5)$$

In Eq. (2),  $\mathbf{V}_{\text{dr},k}$  is the drift velocity for the secondary phase k, i.e. the nanoparticles in the present study.

$$\begin{aligned} \mathbf{V}_{\text{dr},k} &= \mathbf{V}_k - \mathbf{V}_m, \quad \boldsymbol{\tau} = \mu_m \nabla \mathbf{V}_m, \\ \boldsymbol{\tau}_t &= -\sum_{k=1}^n \Phi_k \rho_k \overline{v_k v_k} \end{aligned} \quad (6)$$

The slip velocity (relative velocity) is defined as the velocity of the secondary phase (p) relative to the velocity of the primary phase (f) and the drift velocity is related to the relative velocity

$$\mathbf{V}_{\text{pf}} = \mathbf{V}_p - \mathbf{V}_f, \mathbf{V}_{\text{dr},k} = \mathbf{V}_{\text{pf}} - \sum_{k=1}^n \frac{\phi_k \rho_k}{\rho_m} \mathbf{V}_{fk} \quad (7)$$

The relative velocity is determined from Eq. (7) proposed by Manninen et al. [29] while Eq. (8) by Schiller and Naumann [30] is used to calculate the drag forces

$$\begin{aligned} \mathbf{V}_{\text{pf}} &= \frac{\rho_p d_p^2 (\rho_p - \rho_m)}{18 \mu_f f_{\text{drag}} \rho_p} \mathbf{a} \\ f_{\text{drag}} &= \begin{cases} 1 + 0.15 Re_p^{0.687}, & Re_p \leq 1000 \\ 0.0183 Re_p, & Re_p \geq 1000 \end{cases} \end{aligned} \quad (8)$$

The acceleration (a) in Eq. (8) is given by:

$$\mathbf{a} = \mathbf{g} - (\mathbf{V}_m \cdot \nabla) \mathbf{V}_m \quad (9)$$

An effective solid viscosity model in terms of solid volume fraction was obtained from the experimental work of Millerand Gidaspow [31]:

$$\mu_s = -0.188 + 573.42 \Phi \quad (10)$$

Turbulence is modeled with the Launder and Spalding [32] k-ε turbulence model for the mixture. It is expressed by Eqs. (11) and (12)

$$\nabla \cdot (\rho_m \mathbf{V}_m k) = \nabla \cdot \left( \frac{\mu_{t,m}}{\sigma_k} \nabla k \right) + G_{k,m} - \rho_m \varepsilon + G_b \quad (11)$$

$$\begin{aligned} \nabla \cdot (\rho_m \mathbf{V}_m \varepsilon) &= \nabla \cdot \left( \frac{\mu_{t,m}}{\sigma_\varepsilon} \nabla \varepsilon \right) + \frac{\varepsilon}{k} (c_1 G_{k,m} \\ &- c_2 \rho_m \varepsilon) \end{aligned} \quad (12)$$

Where

$$\begin{aligned} \mu_{t,m} &= \rho_m c_\mu \frac{k^2}{\varepsilon}, \\ G_{k,m} &= \mu_{t,m} (\nabla \mathbf{V}_m + (\nabla \mathbf{V}_m)^T), \\ c_1 &= 1.44, c_2 = 1.92, c_\mu = .09, \sigma_k = 1, \sigma_\varepsilon = 1.3 \\ G_b &= \beta g \alpha_T \mu_t \frac{\partial T}{\partial x_i}, \end{aligned} \quad (13)$$

$\alpha_T$  is inverse effective turbulent prantle number for energy.

The effective properties of the nanofluid (fluid containing suspended nanoparticles) are defined as follows:

$$\rho_m = (1 - \Phi) \rho_f + \Phi \rho_p \quad (14)$$

Chon et al. [5] correlation, which considers the Brownian motion and mean diameter of the nanoparticles, has been used for calculating the effective thermal conductivity

$$\begin{aligned} \frac{k_{\text{eff}}}{k_f} &= 1 + 64.7 \times \Phi^{0.746} \left( d_f / d_p \right)^{0.369} \\ &\times \left( k_p / k_f \right)^{0.746} \times \text{pr}^{.9955} \times \text{Re}^{1.2321} \end{aligned} \quad (15)$$

Where Pr and Re in Eq. (15) are defined as

$$\begin{aligned} \text{pr} &= \frac{\mu_f}{\rho_f \alpha_f}, \text{Re} = \frac{\rho_f B_c T}{3 \pi \mu^2 l_{\text{bf}}}, \mu = A \times 10^{\frac{B}{T-C}}, \\ C &= 140, B = 247, A = 2.414 \text{e} - 5 \end{aligned} \quad (16)$$

$l_{bf}$  is the mean free path of water and  $B_c$  is the Boltzman constant ( $B_c=1.3807 \times 10^{-23}$  J/K).

Effective viscosity is calculated by the following equation proposed by Masoumi et al. [33] which considers the effects of volume fraction, density, and average diameters of nanoparticles and physical properties of the base fluid:

$$\mu_{eff} = \mu_f + \frac{\rho_p \times V_b d_p^2}{72C \delta}, \delta = \sqrt[3]{\frac{\pi}{6\Phi} d_p},$$

$$V_B = \frac{1}{d_p} \sqrt{\frac{18k_b T}{\pi \rho_p d_p}},$$

$$C = \mu_f^{-1} [(c_1 d_p + c_2)\Phi + (c_3 d_p + c_4)]$$

$$c_1 = -0.000001133, c_2 = -0.000002771$$

$$c_3 = 0.00000009, c_4 = -0.000000393$$
(17)

The boundary conditions are expressed as follows:

- At the tube entrance ( $Z = 0$ ):

$$V_z = V_0, \quad V_\phi = V_r = 0, \quad T = T_0,$$

$$I = I_0$$
(18)

Since the adopted model incorporates the assumption of turbulence isotropy, the corresponding turbulent kinetic energy is :

$$k_0 = 1.5(I_0 V_0)^2$$
(19)

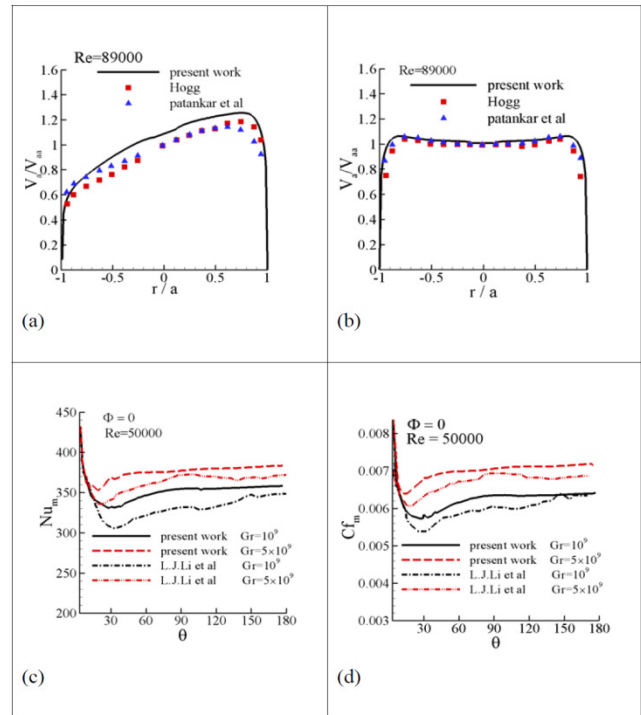
- At the tube outlet:  
The diffusion fluxes for all dependent variables are set to zero at the exit. And an overall mass balance correction is obeyed.
- At the fluid wall interface ( $r = D/2$ ):  
 $V_r = V_\phi = V_z = 0, K = \epsilon = 0, T = T_w$

### 3. Numerical method and validation

This set of coupled nonlinear differential equations is discretized with the Finite volume technique. For the convective and diffusive terms the second order upwind method is used while the SIMPLEC procedure is introduced for the velocity–pressure coupling. A structured non-uniform grid distribution has been used to discretize the computation domain. It is finer near the tube entrance and near the wall where the velocity and temperature

gradients are large. Several different grid distributions have been tested to ensure that the calculated results are grid independent. The selected grid for the present calculations consisted of 160, 48 and 48 nodes, respectively, in the axial, radial and circumferential directions.

In order to demonstrate the validity and also precision of the model and the numerical procedure, calculated axial velocity profile with respect to planes A-A and B-B (see Fig1) are compared with the corresponding experimental and numerical results [25,26]. As it is shown in Fig. 2 (a,b), good agreements between the results are observed. In addition, Fig.2(c,d) shows the comparison of peripherally averaged friction coefficient and Nusselt number with the results of Li et al [27]. Again acceptable agreement between the results is seen.



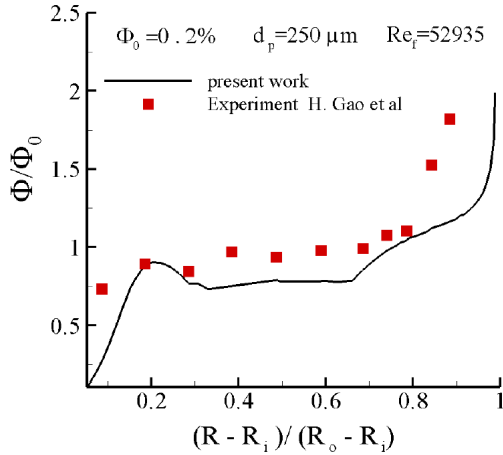
**Fig.2.** Comparison of the calculated results with the experimental and numerical results.

Furthermore, Fig.3 compares the particle concentration profiles with the experimental data [34] in order to verify the two phase model. The root mean square error for this data is calculated by:

$$e_{rms} = \left( \frac{1}{N} \sum_{i=1}^N \left[ \left( \frac{q_{e,i} - q_{d,i}}{q_{d,i}} \right)^2 \times 100 \right] \right)^{\frac{1}{2}} \quad (21)$$

$e_{rms}$  is equal to 6.47%.

Therefore the numerical procedure is reliable and can predict the effect of nanoparticles size on developing turbulent mixed convection flow in a horizontal curved tube.



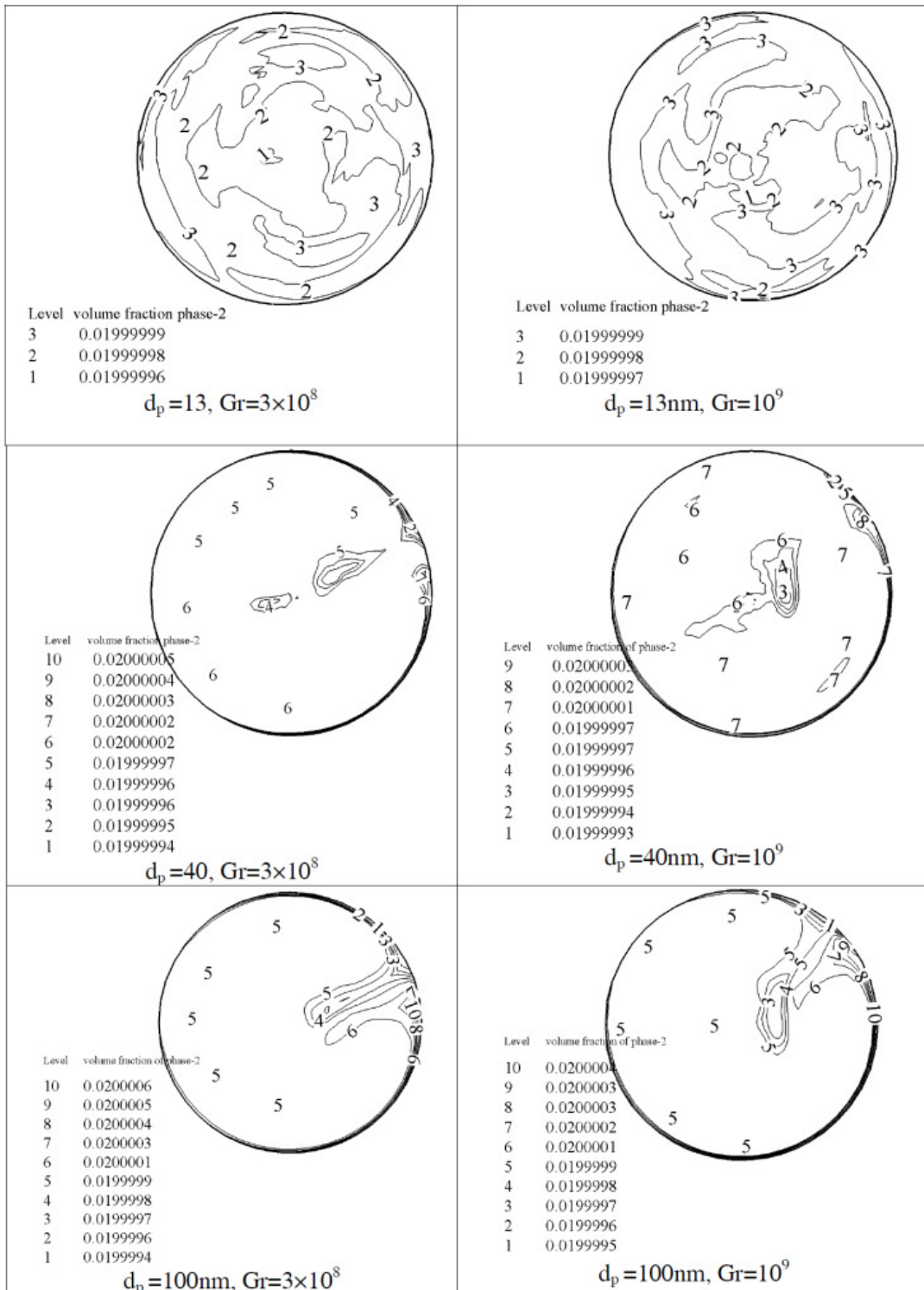
**Fig.3.** comparison of the particle concentration profiles.  $R_i$  and  $R_o$  represent tube inner and outer radius.

#### 4. Results and discussions

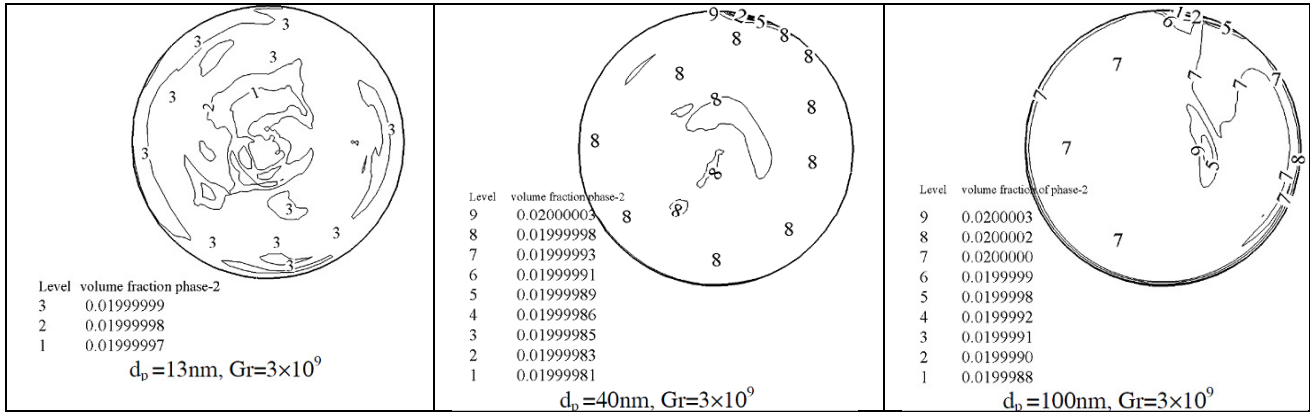
Simulations have been performed on a wide range of  $Re$  and  $Gr$  for five values of particles concentrations. However, because of similar behaviors the results presented here are those for a given  $Re$  and three  $Gr$  at  $\Phi=2\%$ . Three values of particles mean diameter (13nm, 40nm, 100nm) are also considered. Fig 4 shows the Vectors of non-dimensional secondary flow ( $v_r D/\alpha_{eff}$ ,  $v_\phi D/\alpha_{eff}$ ). These figures are oriented in the same manner as shown in Fig. 1 with the inner, top, outer and bottom corresponding to  $\phi=0^\circ$ ,  $\phi=90^\circ$ ,  $\phi=180^\circ$  and  $\phi=270^\circ$  respectively. Fig. 4 shows that, increasing the diameter of solid particles reduces the secondary flow. Increasing the diameter of particles augments the mass of particles. Therefore, the centrifugal force and the buoyancy force cannot displace easily particles. As a consequence, for a particular centrifugal force and buoyancy force the secondary decreases. In addition this may arise from the fact that for small nanoparticle diameters a higher wall temperature is needed in order to keep the Grashof number constant. In general, small nanoparticle slightly increases the secondary flow

strength. However, the structure of the secondary flow does not significantly change with the nanoparticle diameter. Fig 5,6 illustrate the contours of nanoparticles distribution. The particles dispersed uniformly throughout the cross section of tube. However non-uniformity could be seen for large nanoparticles and/or high value of the Grashof number. In general, the nanoparticles concentration is lower at the top of inner wall vicinity and is higher at the outer wall for which the viscous forces are important. At the proximity of top inner wall of the tube, the nanoparticles concentration decreases as a result of the secondary flow at the tube cross section. Centrifugal force causes the particles to move to the outer bend. For the particles with small mean diameter, this variation is not significant and thus uniform particles distribution could be considered. While, increasing the nanoparticles mean diameter, non-uniformity on the particles distribution becomes important. Increasing the Grashof number intensifies non-uniformity of the particles distribution. It does not affect the nanofluid with a small mean diameter.

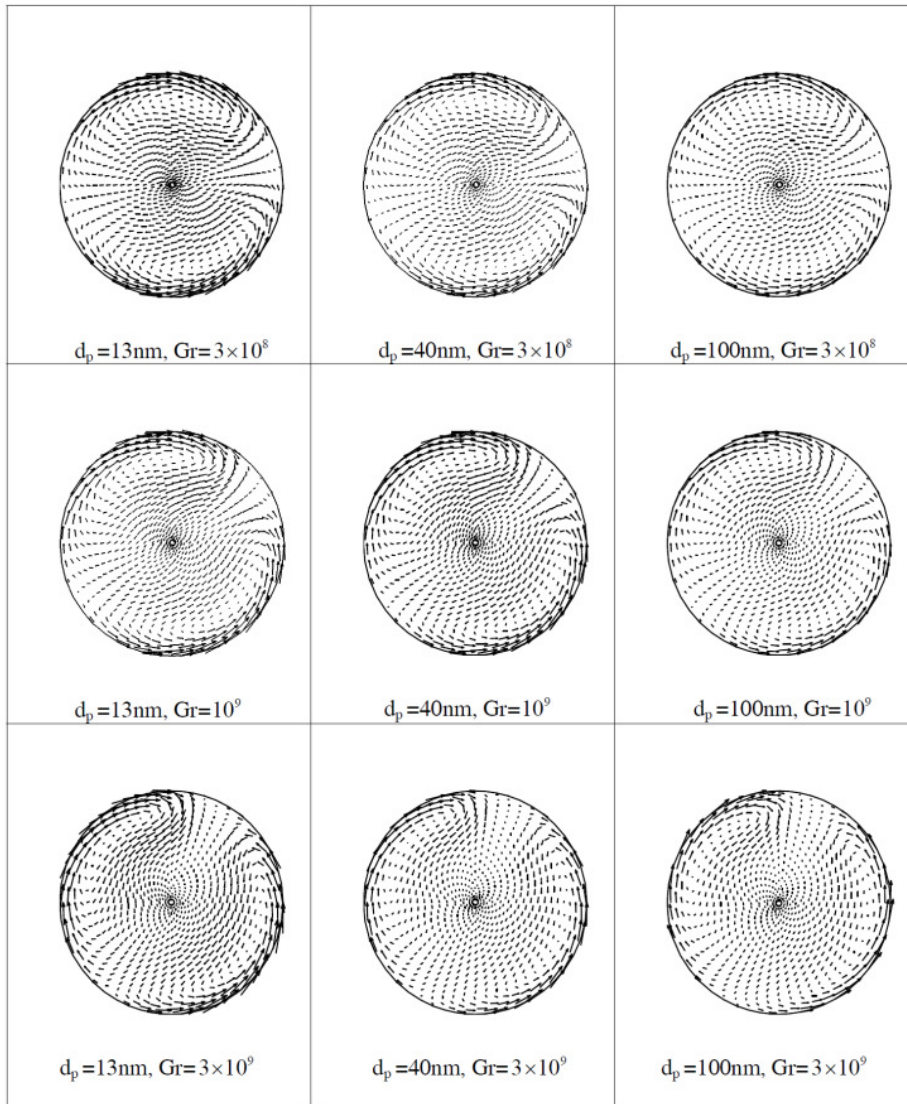
It should be mentioned that the correlations that have been used to calculate the particles relative velocity are those that were developed for micro and milli size particles. However, it could qualitatively show particles distribution. As results of different forces throughout the tube. The effects of nanoparticle diameter on the variations of the turbulence intensity with respect to the horizontal and vertical diameter are presented in Fig. 7. As the particle size decreases the turbulent intensity increases since the smaller particles could be more affected by the secondary flows. This is also understood by looking to Fig.8. The particle concentration profiles on the horizontal diameter for different nanoparticle diameter is presented in Fig 8. In general, the non-uniformity of the nanoparticle distribution increases with decreasing the particle mean diameters. These variations show that the nanoparticles are significantly moved at the tube wall vicinity and the tube centerline region. At the tube wall vicinity the effect of viscous forces could play an important effect on the particle with higher mean diameters. While the tube centerline region is approximately the center of recirculation cells for both the centrifugal secondary induced flow and the buoyancy induced secondary flow.



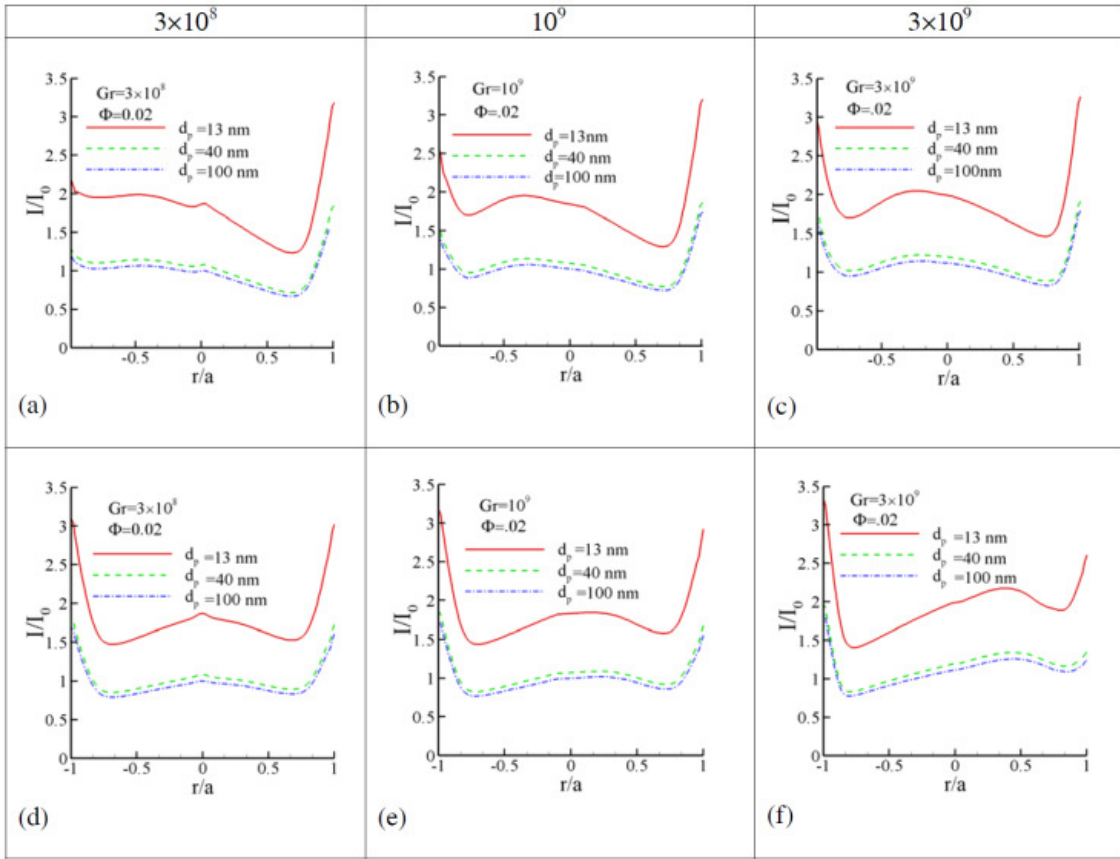
**Fig.4.** Vectors of secondary flow for different  $Gr$  and  $d_p$  at  $Re = 50000$



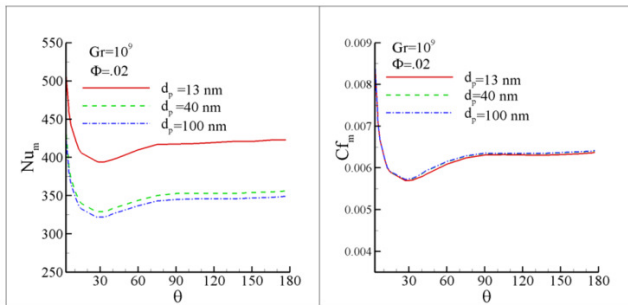
**Fig. 5.** Contours of nanoparticles distribution at  $\theta=90$  degree,  $Re=50000, Gr=3 \times 10^8$  and  $Gr=10^9$



**Fig. 6.** Contours of nanoparticles distribution at  $\theta=90$  degree,  $Re=50000, Gr=3 \times 10^9$



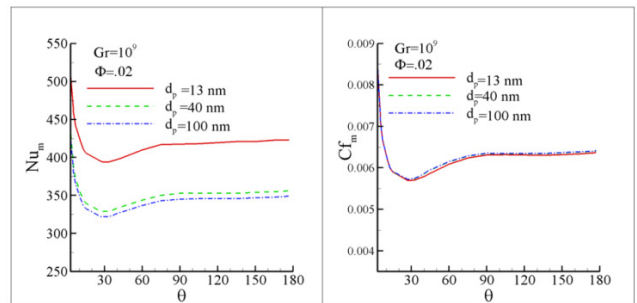
**Fig 7.** Effects of nanoparticle diameter on the turbulent intensity at different Gr on the horizontal diameter (a,b,c) and on the vertical diameter (d,e,f) at  $\theta=135$  degree.



**Fig.8.** The particle concentration profiles on the horizontal diameter (a,b) and on the vertical diameter (d,e) at  $\theta=135$  degree for different nanoparticle diameter.

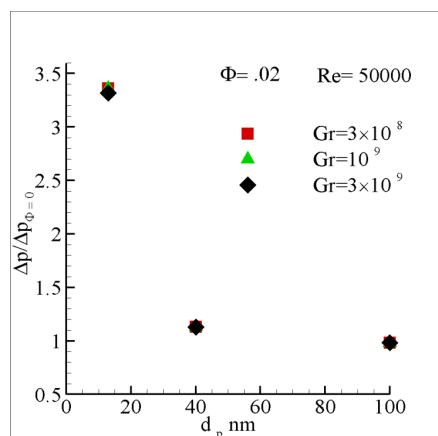
Fig.9 shows the development of the peripherally averaged  $Cf_m$  and  $Nu_m$  along the tube curve at different particle size. The Nusselt number increases as the nanoparticle mean diameter decreases. Smaller nanoparticle can move easily with turbulence eddies. The contact surfaces

between solid particles and fluid increases with decreasing the mean diameter of particles. Thus more heating energy could transfer and consequently the Nusselt number augments. Furthermore, decreasing the diameter of particles enhances the secondary flow.



**Fig.9.** Developing the peripherally averaged  $Cf_m$  and  $Nu_m$  at different particle size along the tube length.

The effects of diameter of particles on the skin friction coefficient are shown in Fig 9b. The effect of nanoparticle mean diameter on the skin friction coefficient is shown in Fig 9.b. As seen there is not any significant effect of nanoparticle mean size on the skin friction coefficient. In general, the skin friction decreases at the tube entrance and then increase as the near wall velocity gradient augments. To this point it should be added that the skin friction is proportional to inverse of inlet velocity square. On one hand, in order to have constant Re at lower diameter, the inlet velocity must increase. On the other hand, at high diameter because of the contacts between solid particles and wall the wall shear stress may augment. There seems to be some scepticism on this regards. However, in order to overcome this problem, Fig 10 shows the normalized pressure drop with different nanoparticle size at three different Grashof number. As seen the pressure drop significantly increases for the small nanoparticles.



**Fig.10.** Effect of nanoparticle size on Nondimensional pressure drop at different Gr .

## 5. Conclusion

The effects of nanoparticles mean diameter have been investigated on the thermo-fluid behavior of turbulent mixed convection of a nanofluid numerically using two-phase mixture model. It is shown that the non-uniformity of nanoparticles distribution throughout the base fluid increases with the nanoparticles mean diameter. Decreasing the nanoparticles mean diameter increases the turbulent intensity. The latter could influence the heat transfer coefficient and the fluid pressure

drop. It could be said that both the heat transfer coefficient and the pressure drop increases with the nanoparticle mean diameter.

## References

- [1] S.U.S. Choi, Enhancing thermal conductivity of fluid with nanoparticles, developments and applications of non-Newtonian flow, ASME FED 231/MD 66(1995) 99–105.
- [2] S. U. S. Choi, Z. G. Zhang, W. Yu, F. E. Lockwood, and E. A. Grulke, Anomalous thermal conductivity enhancement in nanotube suspensions Appl. Phys. Lett 79, (2001) 2252–2255
- [3] H. Zhu, C. Zhang, S. Liu, Y. Tang, and Y. Yin, Effects of nanoparticle clustering and alignment on thermal conductivities of Fe<sub>3</sub>O<sub>4</sub> aqueous nanofluids. Appl. Phys. Lett. 89, (2006) 023123
- [4] S. M. S. Murshed, K. C. Leong, and C. Yang, Investigations of thermal conductivity and viscosity of nanofluids Int. J. Therm. Sci. 47, (2008) 560–568
- [5] C.H. Chon, K.D. Kihm, S.P. Lee, S.U.S. Choi, Empirical correlation finding the role of temperature and particle size for nanofluid (Al<sub>2</sub>O<sub>3</sub>) thermal conductivity enhancement, Appl. Phys. Lett. 87 (2005) 1–3.
- [6] J. Koo and C. Kleinstreuer, A new thermal conductivity model for nanofluids, Journal of Nanoparticle Research 6 (2004), 577–588
- [7] N. Sohrabi, N. Masoumi, A. Behzadmehr, and S.M.H. Sarvari, A Simple Analytical Model for Calculating the Effective Thermal Conductivity of Nanofluids, Heat Transfer Asian Research, 39 (2010) 141–150.
- [8] K. Khanafer, K. Vafai, M. Lightstone, Buoyancy-driven heat transfer enhancement in a two dimensional enclosure utilizing nanofluids. Int. J. Heat Mass Transfer 46 (2003) 3639–3653.
- [9] M. Akbari, A. Behzadmehr, Developing laminar mixed convection of a nanofluid in a horizontal tube with uniform heat flux. Int. J. Heat Fluid Flow 29 (2007) 566–586.
- [10] M. Izadi, A. Behzadmehr, D. Jalali-Vahida, Numerical study of developing laminar forced convection of a nanofluid in an annulus, Int. J. Thermal sciences 48 (2009) 2119–2129.
- [11] Y.M. Xuan, W. Roetzel, Conceptions for heat transfer correlation of nanofluids. Int. J. Heat Mass Transfer 43 (2000) 3701–3707.
- [12] A. Mokmeli, M. Saffar-Avval, Prediction of nanofluid convective heat transfer using the dispersion model. Int. J. Therm. Sci. 49 (2010) 471–478.

- [13] S. Kumar, S. Kumar Prasad, J. Banerjee, Analysis of flow and thermal field in nanofluid using a single phase thermal dispersion model. *Applied Mathematical Modelling* 34 (2010) 573–592.
- [14] Y.M. Xuan, Q. Li, Investigation on convective heat transfer and flow features of nanofluids. *J. Heat Transfer* 125 (2003) 151–155.
- [15] J. Bungirno, convective transport in nanofluid, *J. of Heat Transfer* 128 (2006), 240–250
- [16] A. Behzadmehr, M. Saffar-Avval, N. Galanis, Prediction of turbulent forced convection of a nanofluid in a tube with uniform heat flux using a two-phase approach. *Int. J. Heat Fluid Flow* 28 (2007) 211–219.
- [17] R. Lotfi, Y. Saboohi, A.M. Rashidi, Numerical study of forced convective heat transfer of Nanofluids: Comparison of different approaches. *Int. Com. Heat Mass Transfer* 37 (2010) 74–78.
- [18] V. Bianco, O. Manca, S. Nardini, Numerical investigation on nanofluids turbulent convection heat transfer inside a circular tube. *Int. J. Thermal. Sci.* 50(2011)341-349
- [19] S.Kondaraju, E. K. Jin, and J. S. Lee. Investigation of heat transfer in turbulent nanofluids using direct numerical simulations, *Physical Review E* 81, (2010) 016304.
- [20] Mirmasomi and behzadmehr, Effect of nanoparticles mean diameter on laminar mixed convection heat transfer of a nanofluid *Int. J. of Heat and Fluid Flow* 29 (2008) 557–566.
- [21] A. Akbarinia, R. Laur Investigating the diameter of solid particles effects on a laminar nanofluid flow in a curved tube using a two phase approach. *Int.J.of Heat and Fluid Flow* 30 (2009) 706–714
- [22] S.Vashisth, V.Kumar, Kirishna D.P.Nigam. A Review on potential applications of curved geometries in process industry. *Ind. Eng .Chem. Res.* 47 (2008), 3291-3337
- [23] S.A.Berger, L.Talbot, S.L.Yao, Flow in curved pipes *Annual Review of Fluid Mechanics* 15(1983) 461-512.
- [24] R.K.Shah, S.D.Joshi. Convective heat transfer in curved ducts. *Handbook of single phase convective heat transfer.* New York: John Wiley and Sons (1987).
- [25] G.W. Hogg, The effect of secondary flow on point heat transfer coefficients for turbulent flow inside curved tubes. PhD Thesis, University of Idaho (1968).
- [26] S.V.Patankar, V.S. Pratap, D.B.Spalding, Prediction of turbulent flow in curved pipes. *J Fluid Mech* 57(1975)583-95.
- [27] L. J. Li, C. X. Lin, M. A. Ebdian. Turbulent mixed convective heat transfer in the entrance region of a curved pipe with uniform wall-temperature. *Int. Journal of Heat and Mass Transfer* 41(1998)3793-3805.
- [28] O.Ghaffari, A.Behzadmehr, H.Ajam., Turbulent mixed convection of a nanofluid in a Horizontal curved tube using a two phase approach. *Int.J. Com. Heat Mass Transfer* 37 (2010) 1551–1558
- [29] M.Manninen, V.Taivassalo, S. Kallio, On the Mixture Model for Multiphase Flow, VTT Publications 288(1996). Technical Research Center of Finland.
- [30] L.Schiller, A. Naumann, A drag coefficient correlation. *Z. Ver. Deutsch. Ing.* 77(1935), 318–320.
- [31] D.Gidaspow, *Multiphase Flow and Fluidization.* Academic Press(1994).
- [32] B.E.Launder, D.B.Spalding, *Lectures in Mathematical Models of Turbulence.* Academic Press(1972), London, England.
- [33] N. Masoumi, N. Sohrabi, A. Behzadmehr, A new model for calculating the effective viscosity of nanofluids, *J. Phys. D Appl. Phys.* 42 (2009) 055501.
- [34] H.Gao, L. Guo, X. Zhang. Liquid–solid separation phenomena of two-phase Turbulent flow in curved pipes. *Int.J. of Heat and Mass Transfer* 45 (2002) 4995–5005