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Investigation and Enhancement for Optimal Smoothing and Filtering of Chaotic and Noisy Dynamical Systems

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ABSTRACT

Adaptive Kalman filtering method is based on generating a separable Variational model for estimating joint posterior distribution of states in dynamical system and noise parameters on each time step separately. In this article we present a Gaussian approximation based framework for optimal smoothing of non-linear stochastic state space models, and also time-varying noisy measurements of the system are obtained at discrete instances of time. It is also shown how the method can be applied to a class of models. The result is a recursive algorithm, where on each step the state is estimated with Kalman filter and the sufficient statistics of the noise variances are estimated. We also numerically compare accuracies and error performance of the algorithm with different simulated data.

INTRODUCTION

Continuous discrete filtering models are common in engineering applications especially in the field of navigation, control and communications [1]. In these systems a physical phenomenon can be modeled as a stochastic differential equation, which is observed at discrete instances of time with certain physical sensors. The objective of filtering or recursive estimation is to obtain the dynamical system state from the observed noisy measurements. The methods are based on modeling the transition densities of the processes and modeling the unobserved sample information in data. Gaussian optimal filtering or assumed density filtering with Gaussian assumption is a well-known method, where the filtering distribution is assumed Gaussian [2]. And also the variational Bayesian to linear dynamical systems also known as linear – Gaussian state-space model is important model in control engineering. These models are extensively used in the field of prediction and control and signal processing, because many systems of our interest can be modeled and approximated using linear systems and these models are easier than nonlinear models and also linear systems can be estimated from data efficiently. In state-space models, we assume that the observed time series of data was generated from an underlying sequence of unobserved variables that evolve with Markovian dynamics across successive time steps.Then, the filtering task tries to infer the likely values of the hidden variables that the generated and derive the current observation but in prediction case it tries to simulate the unobserved dynamics one or many steps in our simulation to predict a future observation.

The kalman filter [1] considers estimation of the dynamic state of our model from noisy measurements where the dynamic and measurement can be modeled by linear Gaussian state space models. But in other words the extended kalman filter (EKE) and unscented kalman filter (UKF) extend this method to non-linear dynamic and measurement in our system by forming a Gaussian approximation to the posterior state distribution [2]-[5]. These methods have a limitation in that they assume complete a priori knowledge of the measurement and dynamic model parameters, including the noise statistics. However, the exact knowledge of the parameters of the dynamical systems, especially the noise statistics, is often not a plausible assumption in many practical control applications. Examples of such applications include positioning systems or fault-tolerant systems [6], [7].

Generally, The whole classical noise adaptive filtering approaches can be divided into maximum likelihood, correlation, Bayesian and covariance matching methods [8].but the Bayesian method is the most general and the other methods can sometimes be interpreted as Bayesian approach. Examples of this method are described well in [9], [10].

And also A general variational version of the Kalman smoother has been developed and completely described in

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[11], but with stationary noise. Time varying variance models have been modelled in [12] by using the fixed form approach. The purpose of these methods is to compute the posterior distribution of the current state in dynamical systems.

Our proposed method approximates the joint posterior distribution of the state in dynamical system and the noise variances by a free form distribution. This approximation is formed on each time step separately, and the result is a recursive algorithm, where on each step time the sufficient statistics of the state and the noise variances are estimated and modelled with a fixed-point iteration of a Kalman filter. And also in this algorithm a heuristic model for the variances, which can be used for modelling the time behavior of the noise, is proposed and complete detail of this approached can be find in [3].

PROBLEM FORMULATION

For example The discrete-time linear state space model considered in one of the simulation is:

$$x_k = A_k x_{k-1} + q_k \quad (1)$$

$$y_k = H_k x_{k-1} + r_k \quad (2)$$

Where q_k~N(0, Q) is the Gaussian process noise, and r_k~ N(0, Σ_k) is the measurement noise with diagonal covariance Σk, and the initial state has a Gaussian prior distribution x0 ~ N(m(0), P(0)). The measurement y_k is a d-dimensional vector and the state x_k is an n-dimensional vector. Time is modelled by k and the matrices A_k, h_k, q_k are assumed to be known[5].

Now, departing from the case of standard Kalman filter, we assume that the observation noise , are stochastic with independent dynamic models in our algorithm. The construction of a suitable dynamical model of the observation noise variances will be discussed in [13] and at this stage we don't go to the detail of relations.

The purpose of optimal filtering of the above model is to compute the posterior distribution $p(x_x, \Sigma_k | y_1 : k)$. Formally, the well-known recursive solution to this filtering problem consists of the following steps respectively:

Initialization that starts from the prior distribution and then prediction and finally update that in this stage the predictive distribution is updated to a posterior distribution.

In the following, the different numerical simulations for different problem in control theory and prediction section is described.

NUMERICAL SIMULATIONS

A. Simple Pendulum with Noise

In this section the continuous-discrete sequential is applied to estimation of a partially measured simple pendulum model which is distorted by a random noise term. The stochastic differential equation for the angular position of a simple pendulum ,which is distorted by random white noise accelerations w(t) with spectral density q can be written as:

$$\frac{d^2x}{dt^2} + a^2 \sin(x) = w(t) \quad (3)$$

Where a is the angular velocity of the pendulum. If we define the state as x = (x dx/dt)^T and change to state space form the model can be written as [7]:

$$\frac{dt_1}{dt} x2 \quad (4)$$

$$dx2 = -a^2 \sin(x1)\, dt + d\beta \quad (5)$$

Assume that the state of the pendulum is measured once per unit time and the measurements are disturbed by Gaussian measurement noise with an unknown variance Σ_2 then a suitable model in this case is:

$$y_k \sim N(x_1(t_k), \sigma^2)$$
$$\sigma^2 \text{Inv} - x^2(v_0, \sigma_0^2) \quad (6)$$

In Fig. 1 we showed this model with measurement data and actual and estimated signal and according to the Fig. 1 it is evident that the estimation of signal is most generated in area with high concentration of data and in this area the correlation of our data is higher so this algorithm can detect this important information for producing and estimation of our signal.

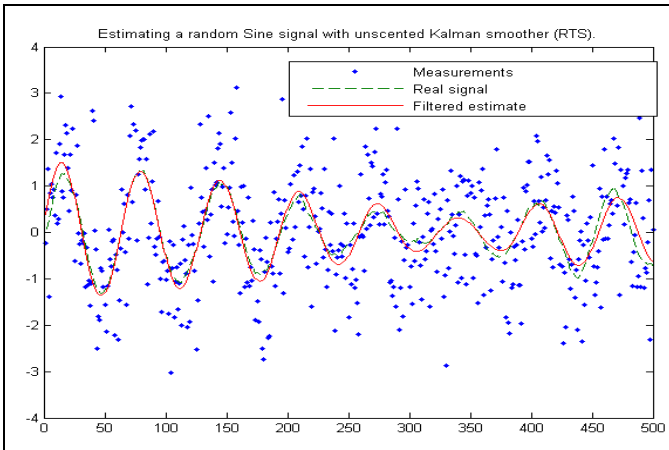


Fig. 1. Filtering results of first order EKF

In these numerical simulations, we use the Extended Kalman Filter as well as the Unscented Kalman Filter to estimate the highly nonlinear time series model. The purpose of this simulation is to demonstrate the potential advantages of the augmented form Unscented Kalman Filter over the non-augmented version. We also compare the performance and accuracy of the different Kalman Filters to a Bootstrap filter, which is a basic particle filter, in modeling the system. In Fig. 2 we have plotted the estimates produced by augmented UKF, first order EKF and Bootstrap filter. For example, during samples 45-60 the EKF clearly follows the wrong mode of our data. The same can be observed from Fig. 3, which contains the absolute errors of the previous estimates and their credible intervals. Figure also shows that EKF is clearly more over confident than UKF and Bootstrap-filter. However, the difference between the two latter methods can't be seen clearly from these two figures. Bycalculating the mean square errors it becomes evidence, that the Bootstrap-filter is clearly superior over other tested methods in modeling of data. This is most likely to the Gaussian approximation made in EKF and UKF is known not to work well in multi-modal cases and it is important factor for limitation of these filters. Another interesting observation can be made from Fig. 4, in which we have plotted the filtering results of a non-augmented form UKF (UKF1) and a augmented one (UKF2). The absolute errors, credible intervals, and estimates for position and velocity produced by the Kalman smoother are plotted in Figures 5 and 6. These figures show the performance and accuracy of each model, and the better performance of each figure is the relative factor that changes with respect to the time series data.

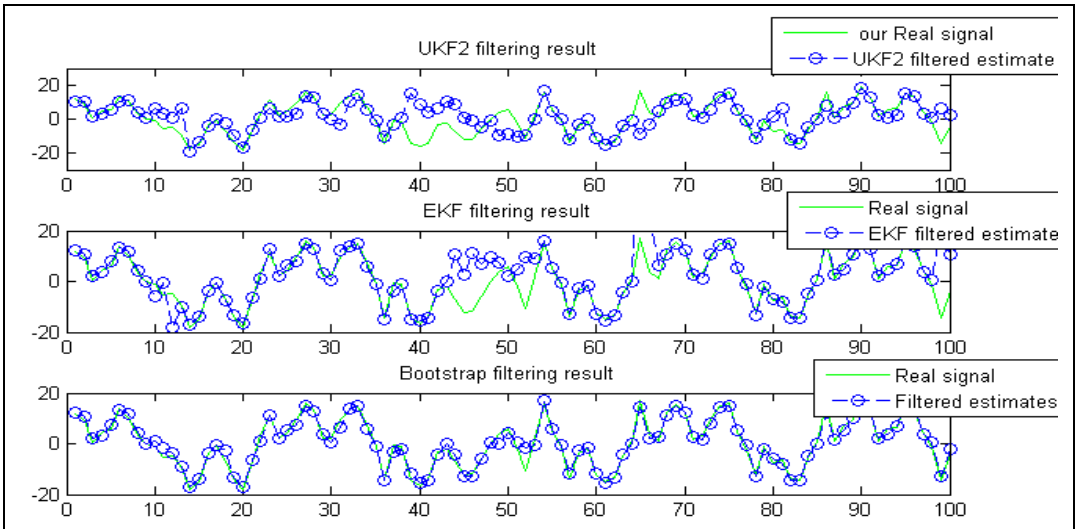


Fig. 2. Estimates for 100 first samples of UNGM-model produced by a) augmented UKF, b) first order EKFand c) Bootstrap filter

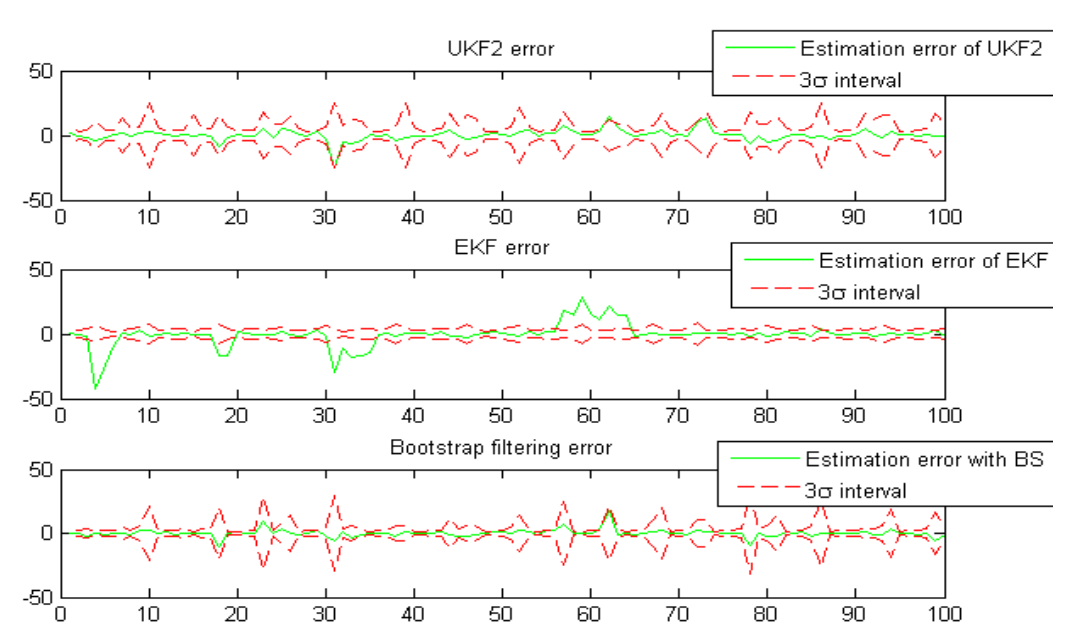


Fig. 3. Absolute errors and credible intervals of previous estimates

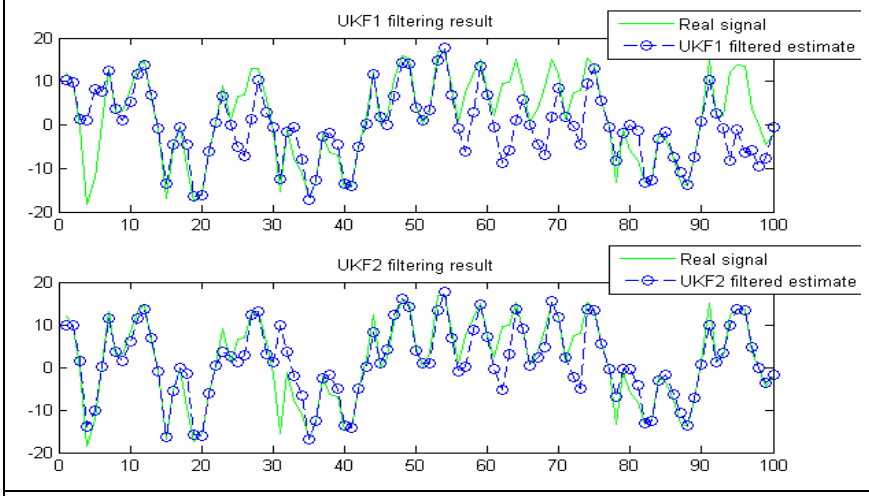


Fig. 4. Estimates for position and velocity produced by Kalman smoother

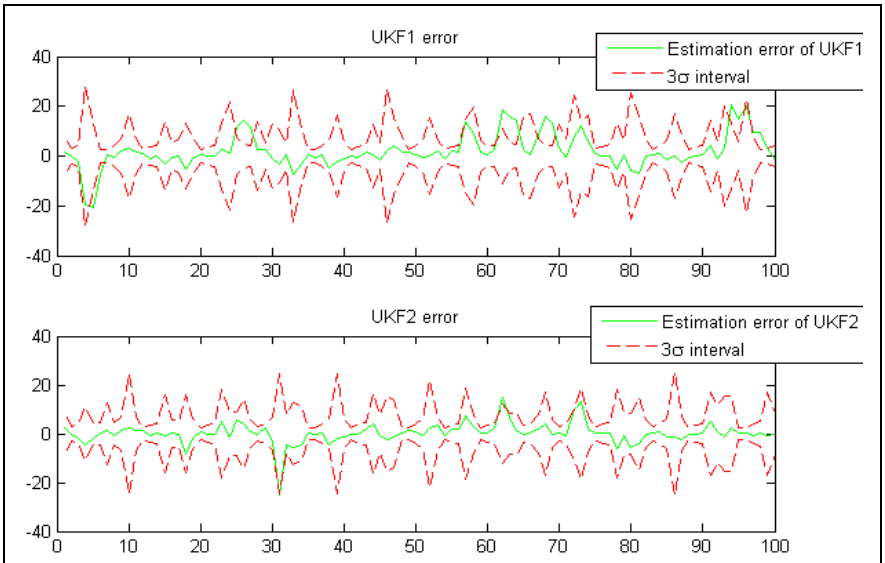


Fig. 5. Absolute errors and credible intervals of previous estimates

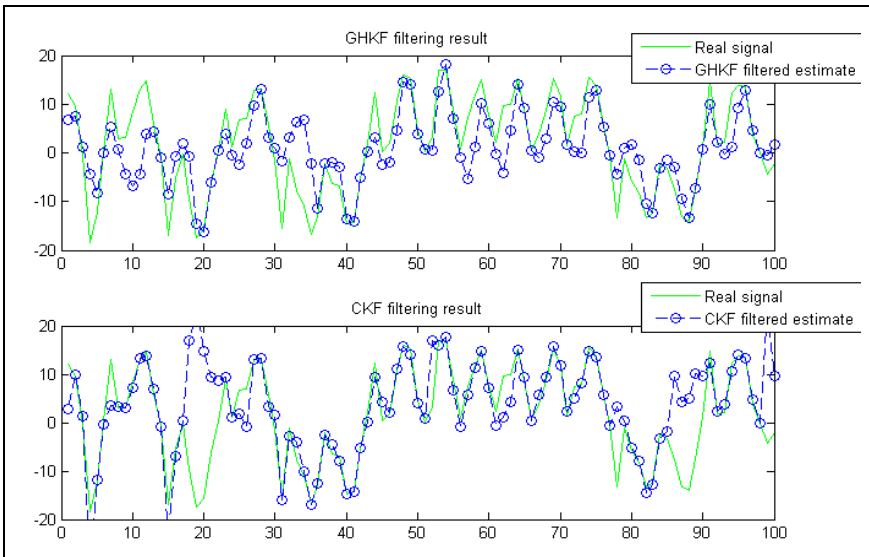


Fig. 6. Estimates for position and velocity produced by Kalman smoother

TRACKING PERFORMANCE OF ALGORITHM

In this simulation we track a moving object with two sensors, that is located according to Fig. 7 and these sensors measure only the angles between the object and them, that is, the measurement model is nonlinear. Fig. 7 shows the measurements made by the sensors in one simulation run. The dynamics of the object are modeled with Wiener velocity model, which means that the velocity of the object is disturbed with Gaussian noise [3]. We have plotted the estimates produced by first and second order EKF and UKF as well as their smoother counterparts. These results for showing the performance of the algorithm is shown relative to real trajectory.

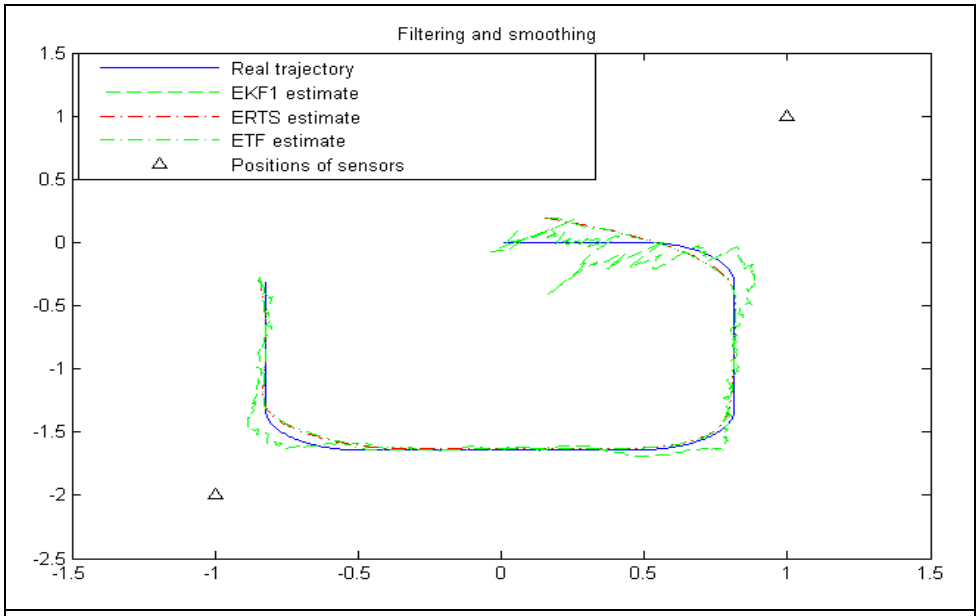


Fig. 7. Filtering and smoothing results of first order EKF

TRACKING A TARGET WITH SIMPLE MANEUVERS

In this simulation filter is used in estimating the trajectory of a target, whose dynamics are modeled with the following two alternating models: the standard Wiener process velocity and acceleration models and the completer model is expressed in [7] and [8]. The purpose of the latter is produce sharp turn to object's trajectory, while the former keeps the trajectory stable in trajectory. Fig. 8 shows the filtering and smoothing results of the filter and smoother using the two models mentioned above. The performance has been compared to the classical Kalman filter and is smoother in Figure 9. The filter provides estimates for model probabilities on each sampling period. Figure 9 also shows the filtered and smoothed probability of the Wiener velocity model at each time step. It is evident that the lag in the filtered estimates is reduced by the application of the smoother.

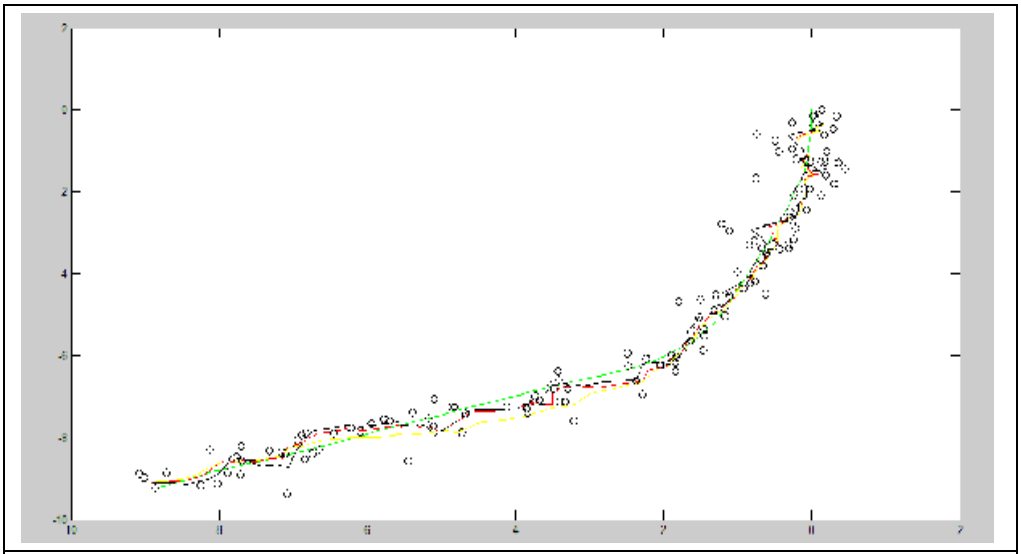


Fig. 8. Filtered and smoothed estimates produced by the filter and smoother

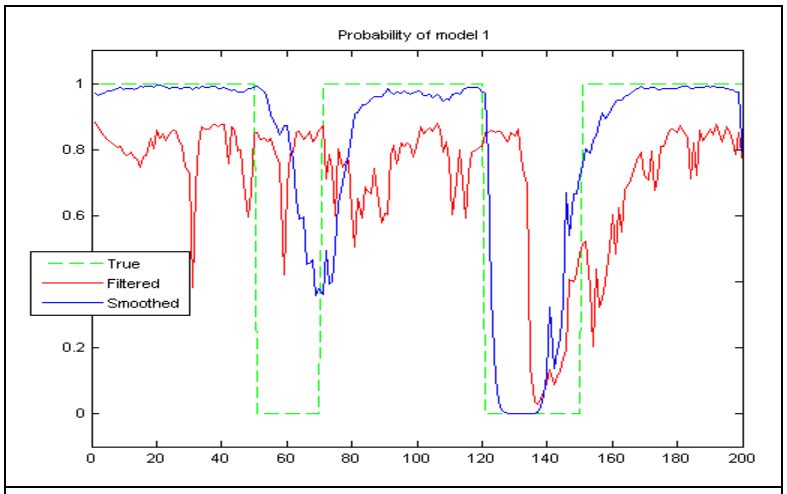


Fig. 9. Estimated probability of the Wiener velocity model

PERFORMANCE OF ALGORITHM IN SWITCHING STATE

In this section, we model three different time series with different distributions, as shown in Figure 10. We then integrate these three-time series, as depicted in Figure 11. The proposed filter is then applied to obtain the probability of changing the state in the dynamical system, which is presented in Figure 12. In this simulation, we find that these algorithms can be used for detecting and modeling switching linear dynamical systems, which is an important factor for detecting changes in the data.

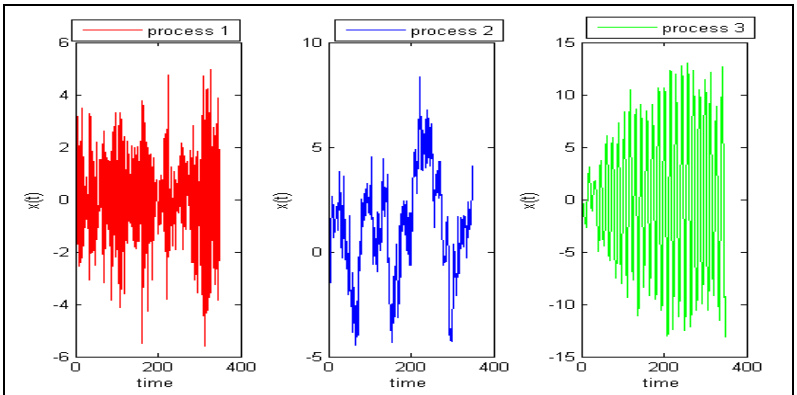


Fig. 10. Three distribution of data

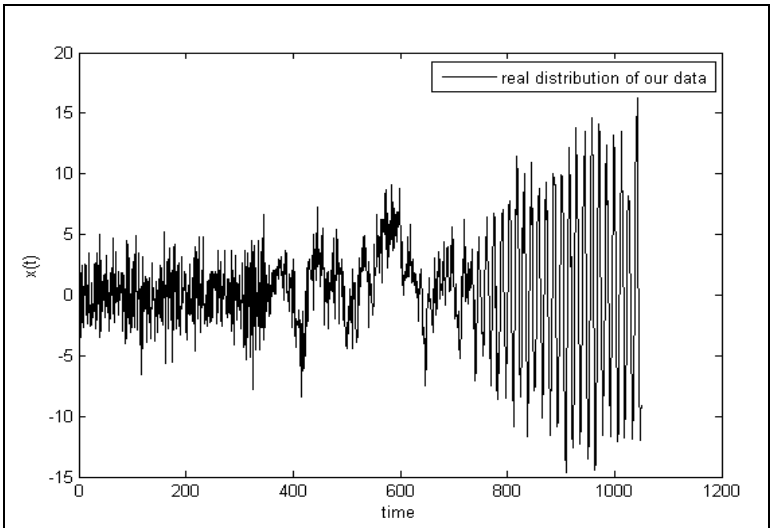


Fig. 11. Integration of three time series

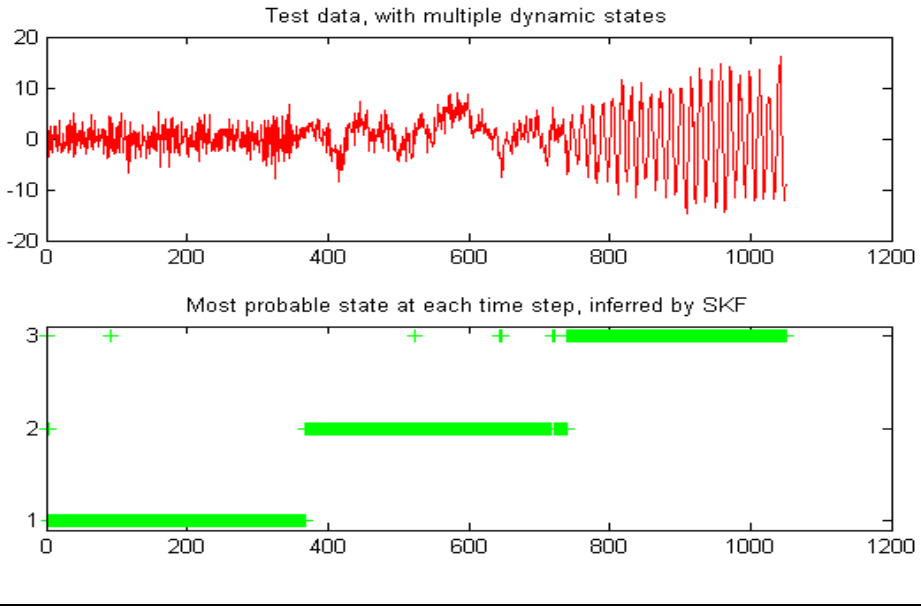


Fig. 12. Probability distribution and detection of switching state

CONCLUSION

In this note, we have presented a new framework for approximate Gaussian optimal smoothing of non-linear state and performance and limitation of different filters for estimation is presented and discussed in different simulation problems.

DECLARATIONS

Data Availability Statement: Not applicable.

Conflicts of Interest: The authors declare no conflict of interest.

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