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ORIGINAL RESEARCH PAPER

Analytical investigation of vibration behavior of damaged micro sensor based on modified couple stress theory

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ABSTRACT: Due to the extensive development and increasing use of microsystems, it is important to predict the behavior of these structures, especially their vibration behavior. In this study, a micro sensor that has been damaged by a crack is investigated. The damaged micro sensor is modeled as a micro beam with a crack. In this modeling, the crack is modeled as a torsion spring. In the modeling, flexoelectric effect, piezoelectric effect, and electric field caused by the applied voltage have been considered. After mathematical modeling, the governing equations have been extracted using Hamilton's principle based on the modified couple stress theory(MCST). The results of the analytical solution of the equations show that increasing the voltage applied to the micro sensor causes the natural frequency to increase and increasing the piezoelectric constant causes the effective stiffness of the micro structure to increase and as a result the natural frequency increases. Also, the results show that changing the flexoelectric constant in the micro sensor does not significantly change the natural frequency of the system.

KEYWORDS*: Crack; Flexoelectric; MCST; Micro beam; Natural frequency; Piezoelectric*

INTRODUCTION

Microelectromechanical systems(MEMS) have advanced greatly due to their small size, lightweight, high performance, easy mass production, and low cost. Due to the development of microsensors, it is very important to investigate the possible damage to this structure. One of the common structural and material damages that affects the dynamic behavior of the microsensor is cracking. The presence of cracks affects the performance of the device and is a possible cause of device failure[1]. The materials used to make micro-sensors have piezoelectric and flexoelectric properties. At the macro scale, flexoelectric properties are weak in contrast to the piezoelectric effect. But at the micro/nano scale, this property is important [2].

Flexoelectric properties may be affected the vibrational behavior of microsystems. Research has been done on the effect of crack and the effect of flexoelectric properties on the dynamic behavior of the micro/nanostructure. Esen et al. [3] Developed a mathematical model to investigate the vibrational behavior of cracked functional micro beam (FG). This micro beam is based on elastic and is subjected to thermal and magnetic fields. They used a torsion spring to model the crack in the beam. Using the Euler-Bernoulli beam theory and the nonlocal elasticity theory, they extracted the equation of motion of the micro beam. They also added the effects of thermal loading and foundation parameters on the cracked microbeam, and obtained the equation of motion of the whole system using the Hamilton principle.

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Sourki and Hoseini[4] investigated the transverse vibration of a cracked microbeam based on the modified couple stress theory. They used the Hamilton principle to derive equations. They modeled the crack with a torsion spring, which adds strain energy from the crack to the system, so the bending slope becomes discontinuous. Akbas[5] analyzed the transverse vibration of a micro-beam consisting of functionally graded material (FGM) with edge cracks based on the modified couple stress theory. He derived the equations of motion using Lagrange equations. He examined the difference between classical and non-classical theory for micro-beam vibration. Akbas [6] investigated the forced vibration of a cracked nano-beam by applying the damping effect. He modeled the damping with the Kelvin–Voigt model. He used the finite element method in Timoshenko beam theory of time in the field of time to solve a dynamic problem. He also compares different beam theories in forced vibration results. Hsu et al. [7] investigated the longitudinal frequency of nano beam with cracks. They extracted the nano beam vibration equations with different boundary conditions and based on the elasticity theory. Their results showed that the frequency decreases with increasing the crack depth. Nazemnezhad and Fahimi [8] studied the free-rotational vibration of a nano-beam with a crack with different boundary conditions. They investigated the effects of surface energy and surface density on vibrational behavior.

The governing equations were derived using the Hamilton principle. The results of their study show that the effect of surface energy on longitudinal and transverse vibration is not the same. Hassannejad and Jahed [9] studied the dynamics behavior of cracked beams in the presence of AC and DC loads. They presented a nonlinear analytical model of the cracked microbeam by applying axial residual stress and fringing field. They considered the crack to be a massless torsion spring. They solved the governing equations using the Galerkin method and the results of their study showed that the behavior of ordinary cracked beams is different from that of cracked micro beams due to nonlinear effects. Shabani and Cunedioglu [10] studied the free vibration of non-uniform beams made of functionally graded material (FGM) with edge cracks. Tymoshenko beam theory was used to analyze the finite element of multilayer sandwich beams. They examined the effect of different crack parameters. Arani et al.[11] investigated the free vibration of a sandwich micro-beam that is exposed to an electric field. They used the Euler-Bernoulli beam theory and the modified strain gradient theory to study micro-beam vibration. The sandwich micro beam is made of two flexo-electric plates and a functionally graded porous core. They used the Hamilton principle to derive the problem equations. They investigated the effects of various parameters such as porosity index, flexoelectric loads, and length scale parameter. Omidian et al.[12] studied the free vibration of Timoshenko nano-beam under the influence of direct and reverse flexoelectric effect. They derived the governing equations and boundary conditions of the Timoshenko nano-beam by considering the direct and inverse flexo-electric effect. After extracting the equations, the effects of flexoelectric coefficient and length effect parameter on natural frequency were investigated. Vaghefpour [13] investigated the nonlinear vibration of an excited nano-actuators made of isotropic dielectric materials with a flexoelectric effect. Vaghefpour derived the nonlinear equation of the Euler-Bernoulli nano- actuators based on non-classical continuous mechanics. He also used the Hamilton principle in deriving the governing equations. He used the sliding mode control algorithm to achieve the desired output for tip tracking. Esmaeili and Tadi Beni [14]. investigated the vibrational and buckling behavior of flexoelectric nano-beams made of functionally graded materials. Using the Hamilton principle, and by considering the Von-Karman strain and forming the enthalpy equation based on displacement, polarization and electric potential, they extracted the equations. The results of their study show that the natural frequency increases with increasing flexoelectric coefficient. Soleimani et al.[15] derived the equations of motion of a sandwich plate based on Love-Kirchhoff hypothesis. In deriving the equations of this plate, they used the Hamilton principle and the modified couple stress theory. The sandwich plate consists of a honeycomb core formed by two plates at the top and bottom of the core. They obtained the equations of both face sheets considering the flexoelectric effect. They examined the effect of material properties and geometric dimensions. Zhang et al. [16] studied the Thickness-Shear (TSh) vibration of a piezoelectric rectangular plate considering the flexoelectric effect. To calculate the flexoelectric effect, they calculated the shear strain gradient in the thickness direction to achieve a simple mathematical model. The results of their study show that the vibration frequencies of the shear thickness states of the thin piezoelectric crystal plate are affected by the flexoelectric property.

Qu et al. [19] proposed a gyroscope with a flexoelectric beam. The flexoelectric beam in the proposed gyroscope simultaneously acts as an actuator and a sensor through two pairs of electrodes and flexural vibrations in perpendicular directions.

Also, they modeled the performance of the proposed gyroscope. Zhang and Zhou [20] investigated the elastic behavior of flexoelectric beam under a magnetic field. They derived the governing equations using the Euler–Bernoulli beam theory and Hamilton's principle. The results of their study show that the applied magnetic field increases the bending stiffness of flexoelectric beam.

In this study, a micro-sensor is modeled as a micro beam. Considering that one of the damages of micro sensors is the possible presence of cracks. The effect of the presence of cracks in the micro sensor has been investigated. For the mathematical modeling of the vibration behavior of the microsensor similar to the real conditions, the effect of the piezoelectric and flexoelectric properties, as well as the existence of the electric field that is the result of applying voltage, have been considered. To investigate the vibration behavior of the micro system, first the equations of the system were extracted by considering the flexoelectric and piezoelectric effect using Hamilton principle based on MSCT and these equations were solved using the separation of variables.

MODELING OF CRACKED MICRO SENSOR

The cracked micro-sensor, which is modeled as a cracked piezoelectric micro- cantilever beam, is shown in Figure 1. The crack is located at a distance from the left support.

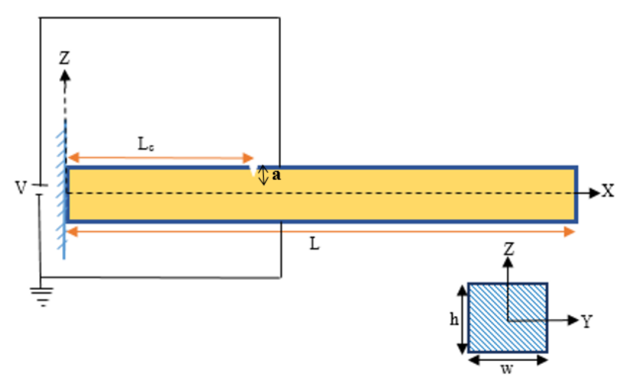

Fig. 1. Micro-cantilever beam with an open edge crack

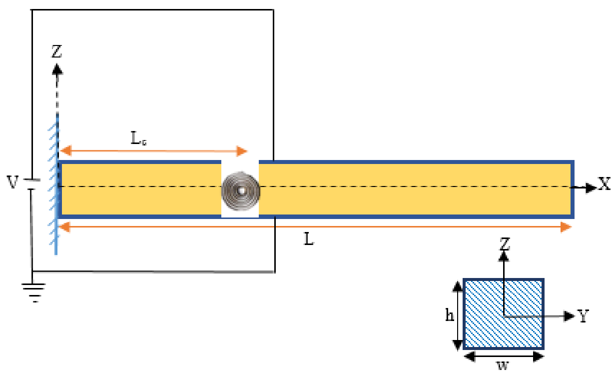

Fig. 2. Modeling of an open edge crack with a torsion spring

Using the references [17] and [18] local crack stiffness for microbeam with flexoelectric and piezoelectric can be calculated as follows:

$$K_t = \left[\frac{EI}{6\pi h(1-\vartheta^2)D}\right] \left[1+\frac{12}{(1+\vartheta)(1-\eta)^2}\left(\frac{l}{h}\right)^2+\frac{e_{31}\mu_{31}}{k_{33}E}+\frac{{e_{31}}^2}{k_{33}E}\right]^{-1} \quad (1)$$

where

$$D = 19.600\eta^{10} - 40.693\eta^9 + 47.041\eta^8 - 33.153\eta^7 + 20.469\eta^6 - 10.092\eta^5 + 4.631\eta^4 - 1.077\eta^3 + 0.629\eta^2 \quad (2)$$

where η is a dimensionless coefficient that represents the ratio of crack depth to beam thickness ($\eta = \frac{a}{h}$). According to Figure 1, the length of the crack is equal to w.

GOVERNING EQUATION

The strain energy of micro beam with flexoelectric and piezoelectric properties can be calculated from equation 3:

$$U = \frac{1}{2}\int_0^l \int_A (\sigma_{xx}\varepsilon_{xx} + 2\,m_{xy}\chi_{xy} + \tau_{xxz}\varepsilon_{xx,z})\, dAdx \quad (3)$$

In this relationship, which is based on the Euler-Bernoulli beam theory, σ_{xx} and ε_{xx} represents stress and strain, respectively. Also in equation 3, m_{xy} and $\chi_{xy}$are expressed to show the deviatoric part of the couple stress tensor and the symmetric curvature. τ_{xxz} is a higher-order stress tensor and $\varepsilon_{xx,z}$ is the gradients of strain. The constituent parts of the strain energy can be calculated by the following equations. The displacement field in Euler-Bernoulli theory is expressed in equation 4.

$$u_x = -\frac{\partial w(xt)}{\partial x} z, u_y = 0, u_z = w(x,t) \quad (4)$$

The displacement field is used to extract the strain.

$$\varepsilon_{xx} = \left(\frac{\partial^2 w}{\partial x^2}\right) z \quad (5)$$

The stress of the piezoelectric Euler-Bernoulli beam is derived from the following equation by considering the flexoelectric property:

$$\sigma_{xx} = -E\left(\frac{\partial^2 w}{\partial x^2}\right) z - \frac{{e_{31}}^2}{k_{33}}\left(\frac{\partial^2 w}{\partial x^2}\right) z + \frac{e_{31}\mu_{31}}{2k_{33}}\left(\frac{\partial^2 w}{\partial x^2}\right) + e_{31}\frac{V}{h} \quad (6)$$

Deviator part of the couple stress tensor and the symmetric curvature for the Euler-Bernoulli beam are expressed as follows:

$$\chi_{xy} = -\frac{1}{2}\frac{\partial^2 w}{\partial x^2}, m_{xy} = 2\mu l^2 \chi_{xy} \quad (7)$$

Electric field, higher-order stress tensor and the gradients of strain for the Euler-Bernoulli beam are extracted as the following equations:

$$E_z = \left(\frac{e_{31}}{k_{33}} z - \frac{\mu_{31}}{2k_{33}}\right)\frac{\partial^2 w}{\partial x^2} - \frac{V}{h} \quad (8)$$

$$\tau_{xxz} = -\mu_{31} E_z \quad (9)$$

$$\varepsilon_{xx,z} = \frac{\partial^2 w}{\partial x^2} \quad (10)$$

In the above equations, E is the Young module, V is the voltage, L is the material length scale parameter, and h is the thickness of the beam. Also, e_{31} , μ_{31}, k_{33} and E_z, are the piezoelectric constant, flexoelectric constant, dielectric constant and the electric field, respectively.

It can be calculated using the following equations of kinetic energy (T) and external work (W):

$$T = \frac{1}{2}\int_0^l \int_A \rho\left(\frac{\partial w}{\partial t}\right)^2 dAdx \quad (11)$$

$$W = \frac{1}{2}\int_0^l N_{xx}\left(\frac{\partial w}{\partial x}\right)^2 dx \quad (12)$$

In equation 12, N_{xx} can be calculated from the following equation:

$$N_{xx} = \int_A \sigma_{xx} dA \quad (13)$$

As shown in Figure 2, the cracked micro-beam can be divided into two parts with separate displacement and time functions, which are connected by a torsion spring at the crack location. According to the above, the equations related to strain, kinetic energy, and external work change as follows.

$$U = \frac{1}{2}\int_0^{l_c}\int_A (\sigma_{xx}\varepsilon_{xx} + 2\, m_{xy}\chi_{xy} + \tau_{xxz}\varepsilon_{xx,z}) dAdx + \frac{1}{2}\int_{l_c}^{L}\int_A (\sigma_{xx}\varepsilon_{xx} + 2\, m_{xy}\chi_{xy} + \tau_{xxz}\varepsilon_{xx,z}) dAdx + \frac{1}{2}K_t\left(\frac{\partial w_2}{\partial x} - \frac{\partial w_1}{\partial x}\right)^2 \quad (14)$$

$$T = \frac{1}{2}\int_0^{l_c}\int_A \rho\left(\frac{\partial w_1}{\partial t}\right)^2 dAdx + \frac{1}{2}\int_{l_c}^{l}\int_A \rho(\frac{\partial w_2}{\partial t})^2\, dAdx \quad (15)$$

$$W = \frac{1}{2}\int_0^{l_c} N_{xx}\left(\frac{\partial w_1}{\partial x}\right)^2 dx + \frac{1}{2}\int_{l_c}^{l} N_{xx}(\frac{\partial w_2}{\partial x})^2\, dx \quad (16)$$

The Hamilton principle states:

$$\int_{t_1}^{t_2}(\delta T - \delta U + \delta W)dt = 0 \quad (17)$$

Using Hamilton principle, the governing equations of motion are derived as follows:

$$\begin{aligned}(\mathrm{EI})_{eff}\left(\frac{\partial^4 w_1}{\partial x^4}\right) + \rho A\frac{\partial^2 w_1}{\partial t^2} - (\mathrm{P})_{eff}\frac{\partial^2 w_1}{\partial x^2} = 0\\ (\mathrm{EI})_{eff}\left(\frac{\partial^4 w_2}{\partial x^4}\right) + \rho A\frac{\partial^2 w_2}{\partial t^2} - (\mathrm{P})_{eff}\frac{\partial^2 w_2}{\partial x^2} = 0\end{aligned} \quad (18)$$

Where

$$(\mathrm{EI})_{eff} = EI + \frac{{e_{31}}^2 I}{k_{33}} + \frac{AEl^2}{2(1+\nu)} + \frac{A{\mu_{31}}^2}{2k_{33}} \quad (19)$$

$$(\mathrm{P})_{eff} = e_{31}Vb \quad (20)$$

The cross section area (A) and inertia moment (I) parameters of the micro beam are defined as follows:

$$\int_A (1,z,z^2)dA = (A,0,I) \quad (21)$$

To solve the solution equations, the following can be done:

$$\begin{aligned}w_i(x,t) = W_i(x)\,.T_i(t)\,; \quad i = 1, 2\\ T_i(t) = A_i\, sin(\omega t) + B_i\, cos(\omega t)\end{aligned} \quad (22)$$

Substituting equation 22 in equation 18 will provide equation 23:

$$\frac{d^4 W_i(x)}{dx^4} - \frac{(\mathrm{P})_{eff}}{(\mathrm{EI})_{eff}}\frac{d^2 W_i(x)}{dx^2} - \frac{\rho A\omega^2}{(\mathrm{EI})_{eff}}W_i(x) = 0 \quad (23)$$

where ω is natural frequency, ρ is the material density.

Assuming $\frac{(\mathrm{P})_{eff}}{(\mathrm{EI})_{eff}} = q$ and $\frac{\rho A\omega^2}{(\mathrm{EI})_{eff}} = \lambda^2$, the general solution of equation 23 is as follows:

$$W_1(x) = C_1\sinh(\alpha_1 x) + C_2\cosh(\alpha_1 x) + C_3\sin(\alpha_2 x) + C_4\cos(\alpha_2 x) \quad ; \quad 0 \le x \le L_c \quad (24)$$

$$W_2(x) = C_5\sinh(\alpha_1 x) + C_6\cosh(\alpha_1 x) + C_7\sin(\alpha_2 x) + C_8\cos(\alpha_2 x) \quad ; \quad L_c \le x \le \mathrm{L} \quad (25)$$

where

$$\alpha_{1,2} = \sqrt{\frac{\pm q + \sqrt{q^2 + 4\lambda^2}}{2}} \quad (26)$$

In equations 24 and 25 C_i , $(i = 1, 2, \ldots, 8)$ are constants, $W_1(x)$ and $W_2(x)$ are the equations of motion for the left and right segments of the crack, respectively.

The micro-beam boundary conditions shown in Figure 2 can be expressed as follows:

$$\begin{aligned}&W_1(0) = 0 \;\; ; \;\; \frac{dW_1}{dx}(0) = 0\\ &\frac{\partial^2 W_2}{\partial x^2}(L) = 0 \;\; ; \;\; \frac{\partial^3 W_2}{\partial x^3}(L) = 0\\ &W_1(L_c) = W_2(L_c) \;\; ; \;\; \frac{\partial^2 W_1}{\partial x^2}(L_c) = \frac{\partial^2 W_2}{\partial x^2}(L_c) \;\; ; \;\; \frac{\partial^3 W_1}{\partial x^3}(L_c) = \frac{\partial^3 W_2}{\partial x^3}(L_c)\\ &\frac{\partial^2 W_1}{\partial x^2}(L_c) = \frac{K_t}{(\mathrm{EI})_{eff}}\left[\frac{\partial W_2(L_c)}{\partial x} - \frac{\partial W_1(L_c)}{\partial x}\right]\end{aligned} \quad (27)$$

If the boundary conditions are placed in equations 24 and 25, 8 algebraic equations are extracted, which if the determinants of the matrix of coefficients of 8 algebraic equations are zero, the natural frequency is obtained. The fields of the matrix of coefficients [Q] are given in Appendix 1.

RESULTS AND DISCUSSION

In this section, the effect of crack geometry and the cantilever micro-beam material characteristics are numerically investigated. The geometrical characteristics and material characteristics of the cantilever micro-beam are shown in Table 1.

Table1
Geometry and material characteristics.

Symbol	Parameters	Values	Units
L	Length of micro beam	800	μm
W	Width of micro beam	300	μm
h	Thickness of micro beam	20	μm
ϑ	Poisson's ratio	0.33	---
E	young's modulus	150	GPa
ρ	Density	7500	kg/m^3
l	Material length scale parameter	5	μm
k_{33}	dielectric constant	0.13	$\mu C/Vm$
e_{31}	piezoelectric constant	-4.35	C/m^2
μ_{31}	flexoelectric constant	10	$\mu C/m$

Figures 3 and 4 show the first and second natural frequencies of the micro-beam at different crack depths at different crack locations, respectively. In this case, the operating voltage is 2 volts. The diagram in Figure 1 shows that the first natural frequency of the system decreases as the depth of the crack increases. Also, with the increase of the crack distance from the beginning of the micro beam, the first natural frequency increases. From the diagram in Figure 3, it can be seen that the effect of the crack depth on the first natural frequency decreases by approaching the end of the cantilever micro beam.

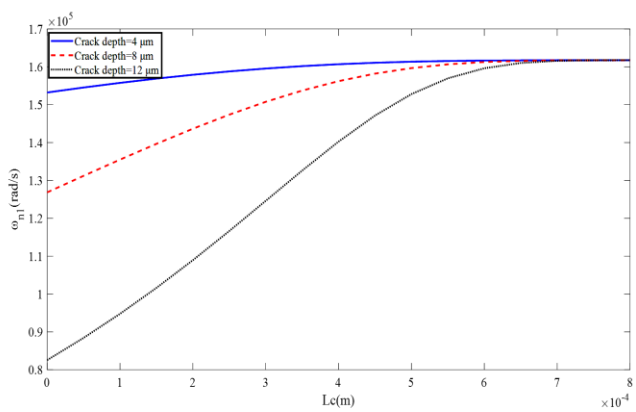

Fig. 3. First natural frequency at three different crack depth

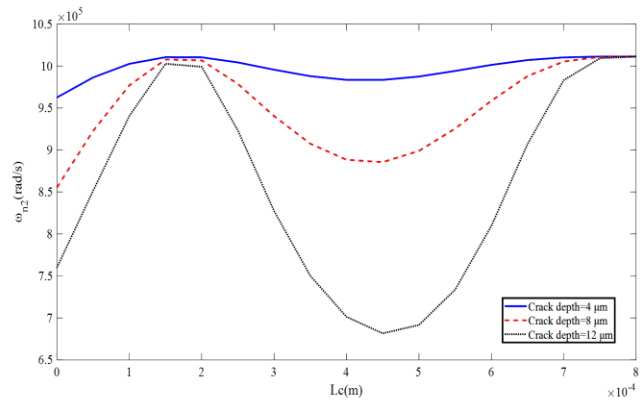

Fig. 4. Second natural frequency at three different crack depth

The diagram in Figure 3 shows the effect of the applied voltage on the cantilever micro beam with piezoelectric and flexoelectric properties.

Figure 5 shows that as the voltage increases, the first natural frequency of the system increases.

In the graphs in Figures 6 and 7, it is possible to see the effect of the piezoelectric constant on the vibration behavior of the cracked microbeam. The graphs show that the first natural frequency increases with the increase in the size of the piezoelectric constant.

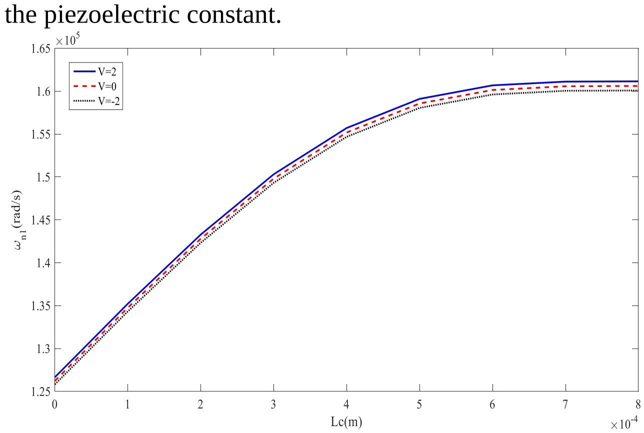

Fig. 5. Effect of voltage on the first natural frequency in 3 different crack depth

This increase is due to the increase in the stiffness of the beam due to the increase in the size of the piezoelectric constant.

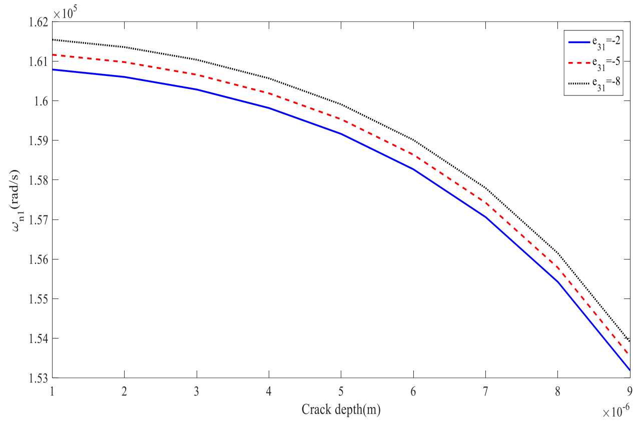

Fig. 6. Effect of crack depth on the first natural frequency in 3 different piezoelectric constant

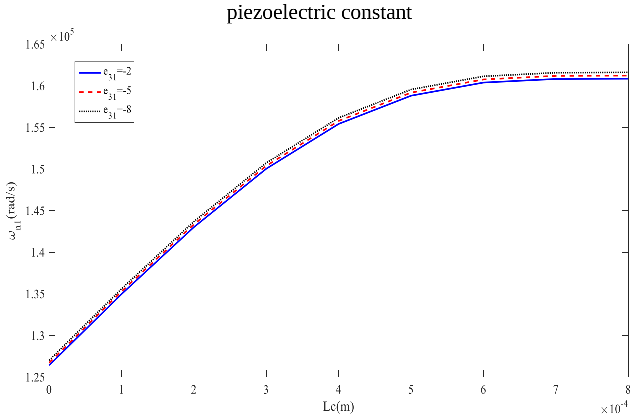

Fig. 7. Effect of crack position on the first natural frequency in 3 different piezoelectric constant

To investigate the effect of flexoelectric on the vibration behavior of the micro-sensor, the diagram of the natural frequency of the micro-beam has been drawn in two different sizes of the flexoelectric constant. This diagram is shown in Figure 8. The diagram shows that changing the flexoelectric constant does not cause a noticeable change in the natural frequency. According to equation 19, the equivalent stiffness consists of 4 terms. Considering that the flexoelectric effect of the second power is a value smaller than one. It is not possible to overcome the other 3 terms and change the size of the equivalent stiffness.

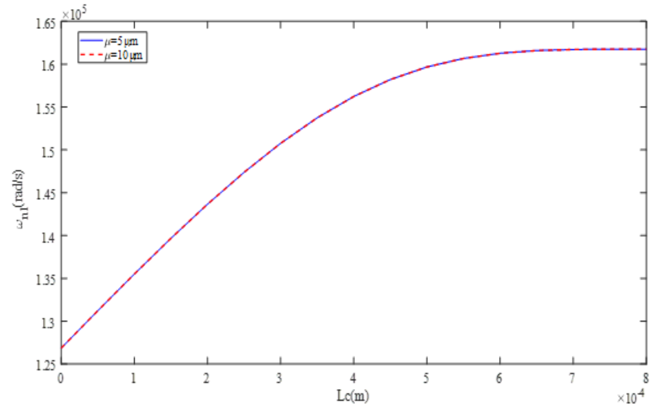

Fig. 8. Effect of crack position on the first natural frequency in two different flexoelectric constant

CONCLUSIONS

In this study, a micro-sensor is modeled as a micro-trigger. In the modeling of the cracked micro beam, a torsion spring is placed instead of the crack, and the stiffness of this torsion spring depends on factors such as crack geometry, micro beam material, and size factor. Considering that the piezoelectric microbeam has piezoelectric and flexoelectric properties. In this study, both flexoelectric and piezoelectric properties have been considered for the stiffness of the torsion spring which is assumed for crack modeling. After deriving mathematical equations using Hamilton's principle based on modified couple stress theory and solving these equations, the following results were obtained:

- An increase in crack depth causes a decrease in the size of the natural frequency of the cracked cantilever micro beam. For example, in the examined beam, doubling the depth of the crack decreases the first natural frequency by about 20%.
- The closer the crack position is to the free end of the beam; the increases the natural frequency of the cracked cantilever microbeam. In the examined beam sample, at a specific crack depth, the first natural frequency will be 50% different if the crack is at the beginning of the beam or at the end of the beam.
- The amount of voltage applied to the micro system has a direct relationship with the natural frequency. That is, as the voltage increases, the natural frequency increases.
- Increasing the piezoelectric constant causes the effective stiffness of the microsystem to increase, and as a result, the natural frequency of the microsystem increases.
- The change in the flexoelectric constant in the micro system does not cause a noticeable change in the natural frequency of the micro system.

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