

## Sensitivity analysis of the effective nanofluid parameters flowing in flat tubes using the EFAST method

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**ABSTRACT:** In the present study, the effective parameters of water- $\text{Al}_2\text{O}_3$  nanofluid flowing in flat tubes are investigated using the EFAST Sensitivity Analysis (SA) method. The SA is performed using GMDH type artificial neural networks (ANN) which are based on validated numerical data of two phase modeling of nanofluid flow in flat tubes. There are five design variables namely: tube flattening ( $H$ ), flow rate ( $Q$ ), wall heat flux ( $q''$ ), nanoparticle diameter ( $dp$ ) and nanoparticle volume fraction ( $\phi$ ) and there are two objective functions namely: pressure drop ( $\Delta P$ ) and heat transfer coefficient ( $h$ ). The results show that among design variables, the tube flattening has the highest effect on variations of pressure drop (74%) and heat transfer coefficient (40%). Except tube flattening, the flow rate and the nanoparticle volume fraction has the highest effect on pressure drop (24%) and heat transfer coefficient (25%) respectively. The effects of all of the design variables on objective functions are shown in the results.

**KEYWORDS:** EFAST method; flat tubes; nanofluid; Sensitivity analysis

### INTRODUCTION

One of the applicable techniques for heat transfer augmentation is the use of Nanofluid. Nanofluid have attracted enormous interest from researchers due to high thermal conductivity and their potential for high rate of heat exchange incurring little penalty in pressure drop. Since a decade ago, research publications related to the use of nanofluid as working fluids have been reported both experimentally and numerically.

Razi et al. [1] investigated a CuO-Oil mixture experimentally in different flat tubes and finally presented correlations for defining the Nusselt number and pressure drop of nanofluid flow in the flat tubes.

Murshed et al. [2] carried out an experimental work, which has been considered the effects of particle size, nanolayer, Brownian motion and particle surface chemistry and interaction potential on the thermal conductivity of nanofluid and proposed a new model for thermal conductivity. Besides experimental studies Computational Fluid Dynamics (CFD) has a great potential to predict the fluid dynamics and heat transfer performance of nanofluid. Shariat et al. [3] used two phase mixture model to simulate the nanofluid flow in different horizontal tubes with elliptical sections.

They simulated laminar and mixed convection flow and discussed the effect of parameters such as Richardson number and particle size on the flow field.

Akbarinia and Laur [4] studied the effect of nanoparticles diameter on nanofluid in horizontal curved tubes with circular cross section using two phase mixture model. Kalteh et al. [5] studied the forced laminar convection nanofluid flow in a rectangular microchannel using two phase Eulerian-Eulerian approach.

Another technique which is employed to enhance heat transfer is the use of flat tubes in heat exchangers. Flat tubes are geometrical modified round tubes which can be manufactured by flattening the round tubes into an oblong shape. Figure 1 shows the schematic of a flat tube cross section.

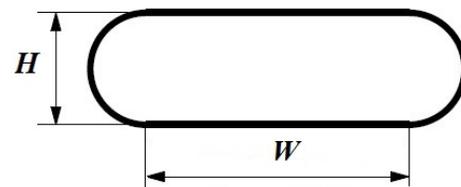


Fig. 1. Geometry cross section of a flat tube

Compared to a circular tube, a flat tube has a higher internal surface area-to-cross-sectional flow area ratio, which can potentially be used to enhance the heat transfer rate and increase the compactness of the heat exchanger. There are only a few articles which have reported the privileges of these kinds of tubes in the favor of heat

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**Table 1**  
Design variables and their range of variations.

Design Variables	From	To
H(mm)	2	10
$Q_i$ (m <sup>3</sup> /hr)	0.002826	0.014130
$q_w''$ (kw/m <sup>2</sup> )	1	5
$\Phi$ (%)	0	5
$d_p$ (nm)	20	100

About the tube flattening (geometry) it should be mentioned that the same perimeter is a constraint which was observed by the other researchers who have studied the flat tubes [1, 8 and 19].

It is evident that since the perimeter of tubes are constant, the H and W parameters in Figure 1 depend on each other and if one of them is known, the other one could be calculated with respect to the constant perimeter constraint. Therefore in this paper, “H” is used to define each flat tube. Figure 2 shows three different flat tubes with the same perimeter constraint.

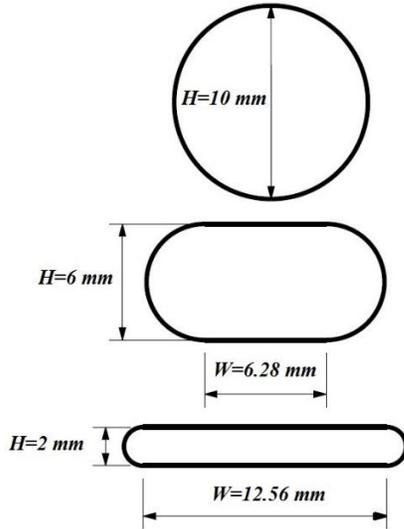


Fig. 2. Dimensions of three flat tube cross sections in the present simulations

Moreover because the hydraulic diameter of tubes are different, the design variables and objective function should not be presented in the non-dimensional form [1,8,19], therefore instead of using non-dimensional parameters such as Nu, Re,  $C_f$  and Gr, their associated dimensional parameters h,  $Q_i$ ,  $\tau_w$  and  $q_w''$  are used. The sensitivity analysis in the present paper is performed using the GMDH type Artificial Neural Network (ANN) models and CFD data which were presented in [9].

**CFD AND GMDH TYPE ANN MODELS**

The sensitivity analysis (SA) presented in this paper is performed using GMDH type artificial neural networks (ANN) which are based on validated numerical data of two phase mixture modeling of nanofluid flow in flat tubes. The

details of numerical modeling and GMDH polynomials are presented in [9]. The details of dimensions and operating conditions are shown in Table 2 and moreover a sample of numerical simulation which presents the details of temperature contours in different flat tubes is shown in Figure 3. In the present study, numerical simulation of nanofluid flow is performed using mixture model which is a single fluid two phase approach. In this method each phase has its own velocity field, and in a given control volume there is a certain fraction of base fluid and nanoparticles. Instead of utilizing the governing equations of each phase separately, it solves the continuity, momentum and energy equations for the mixture, and the volume fraction equation for nanoparticles. A second order upwind method is used for the convective and diffusive terms and the SIMPLE algorithm is employed to solve the coupling between the velocity and pressure fields. Constant velocity and temperature in inlet, constant wall heat flux in walls and fully thermally and hydro dynamically boundary conditions are used.

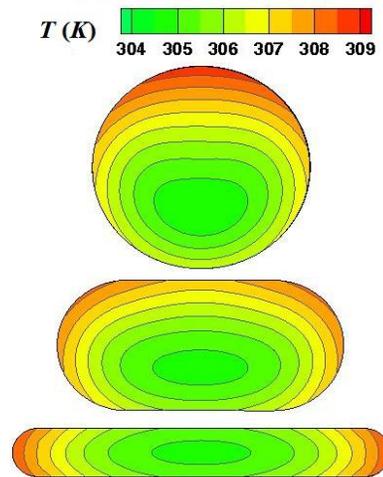


Fig. 3. Effect of tube flattening on fully developed temperature contours

To attain confidence about the simulations, it is necessary to compare the simulation results with the available data.

Figure 4 compares the local heat transfer coefficient for a circular tube of present study with the experimental study carried out by Kim et al. [14] and numerical study of Ebrahimnia et al. [15]. As is evident from this figure the present simulations agree well with the available experimental and numerical data.

**SENSITIVITY ANALYSIS METHODS**

An area of general sensitivity analysis methods that has attracted more attention is the variance-based methods. In these methods, the sensitivity index is computed as the share of each parameter in the overall output variance of the model.

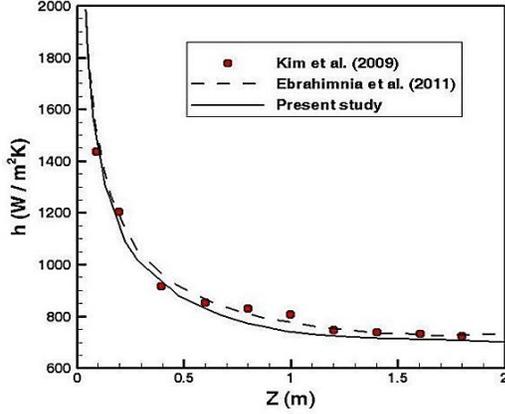


Fig. 4. Comparison of the local heat transfer coefficient with the measured data

The general sensitivity analysis methods are implemented in four steps: (1) defining the inputs and the type of distribution of each input, (2) generating the samples for the input values, (3) computing the model's output for each set of input samples and (4) determining the effect of each input factor on the output [16]. In this section, the variance-based sensitivity analysis methods have been reviewed. The variance-based general sensitivity analysis approaches can be used to obtain the first-order effect and the second-order effect (which include the interaction between other parameters) [17].

The Sobol method [18] is a model-independent general sensitivity analysis method which is based on variance analysis.

This method can be used for nonlinear and non-uniform functions and models. For the model defined by function  $Y=f(X)$ , where  $Y$  is the model output and  $X(X_1, X_2, \dots, X_n)$  is the vector of input parameters, Sobol suggested to decompose the function  $f$  into summands of increasing dimensionality, where the integral of each term over its own input variables is zero.

Sobol showed that, when all the inputs are perpendicular to one another, this resolution is unique and the output variance of the model ( $V$ ) is the set of variances of each resolved term [18]:

$$V(Y) = \sum_{i=1}^n V_i + \sum_{i < j \leq n} V_{ij} + \dots + V_{1\dots n} \quad (1)$$

In relation1,  $V_i$  denotes the first-order effect for each input factor  $x_i$  ( $V_i=V[E(Y|x_i)]$ ), and  $V_{ij}$  ( $V_{ij}=V[E(Y|x_i, x_j)] - V_i - V_j$ ) to  $V_{1, \dots, n}$  indicate the interactions between  $n$  factors. Therefore, the shares allocated to parameters, and the interactions of parameters can be determined from the total output variance.

The sensitivity index is obtained as the ratio of each order's variance to the total variance ( $S_i=V_i/V$ ) denotes the first-order sensitivity index,  $S_{ij}=V_{ij}/V$  represents the second-order sensitivity index, and so on).

The total sensitivity index (i.e., the overall effect of each parameter) is obtained as the summand of all the orders of sensitivity index for that parameter [18]:

$$S_{Ti} = S_i + \sum_{i \neq j} S_{ij} + \dots \quad (2)$$

The EFAST method was presented by Cukier et al. [19] and was later improved by Saltelli et al. [20]. Like the Sobol method, this approach is also based on variance and it is independent of any assumption of linearity and uniformity between inputs and output(s).

Contrary to the Sobol method, which uses multidimensional integrals to obtain the total variance and the partial variances, this method converts the multidimensional integrals to one-dimensional ones by defining a transfer function and simplifies the procedure for the calculation of sensitivity indexes.

The EFAST method searches the  $n$ -dimensional space of the input factors (Unit Hypercube  $K^n$ ) by using a Search Curve defined by a set of parametric equations [20]:

$$x_i = \frac{1}{2} + \frac{1}{\pi} \arcsin(\sin(\omega_i s + \varphi_i)) \quad (3)$$

Where  $\omega_i$  ( $i=1, 2, \dots, n$ ) is the frequency related to factor  $x_i$ ,  $s$  is a variable that changes from  $-\pi$  to  $+\pi$ , and  $\varphi_i$  specifies the starting point of the curve. The output variance of the model is approximated by means of Fourier analysis:

$$V(Y) = \frac{1}{2\pi} \int_{-\pi}^{\pi} f^2(s) ds - \left[ \frac{1}{2\pi} \int_{-\pi}^{\pi} f(s) ds \right]^2 \approx \sum_{j=-\infty}^{\infty} (A_j^2 + B_j^2) - (A_0^2 + B_0^2) \approx 2 \sum_{j=1}^N (A_j^2 + B_j^2) \quad (4)$$

In the above relation,  $f(s)=f(G_1(\sin(\omega_1 s)), G_2(\sin(\omega_2 s)), \dots, G_n(\sin(\omega_n s)))$ ,  $G(s)$  are the transfer functions, and  $A_j$  and  $B_j$  are the Fourier coefficients ( $A_j = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(s) \cos(js) ds$ ,  $B_j = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(s) \sin(js) ds$ ). By calculating the Fourier coefficients for the basic frequency ( $\omega_i$ ) and its higher harmonics ( $p\omega_i$ ), the partial first-order input variance ( $x_i$ ) can be obtained.

$$V_i = \sum_{p \in Z^0} (A_{p\omega_i}^2 + B_{p\omega_i}^2) = 2 \sum_{p=1}^{\infty} (A_{p\omega_i}^2 + B_{p\omega_i}^2) \quad (5)$$

Also, like the Sobol method, the ratio of the first-order partial variance to total variance is used to compute the main sensitivity index.

The total sensitivity index is obtained from relation6 [21]:

$$ST_i = 1 - \frac{V_{-i}}{V} \quad (6)$$

Variance  $V_{-i}$  is obtained by changing all the parameters except parameter  $x_i$ .

The Sobol method employs the Monte Carlo integral to obtain each partial variance; and in comparison with the EFAST method, it doesn't use a transfer function; that's why, it has a low computational efficiency. Algorithm of sensitivity analysis is shown in Figure 5.

### RESULTS OF SENSITIVITY ANALYSIS

The results of sensitivity analysis for heat transfer coefficient ( $\bar{h}$ ) and the pressure drop ( $\Delta P$ ) have been presented in this section. Employing the EFAST method, the sensitivity of five parameters: tube flattening (H), inlet volumetric flow rate ( $Q_i$ ), wall heat flux ( $q_w^*$ ), nanoparticle diameter ( $d_p$ ) and nanoparticle volume fraction ( $\Phi$ ), have been explored for heat transfer coefficient ( $\bar{h}$ ) and the pressure drop ( $\Delta P$ ).

Figure 6a illustrates the changes of the pressure drop with tube flattening. It indicates that, with the increase of this sensitive parameter, the pressure drop diminishes.

As is observed in this figure, at low values of tube flattening, sensitivity is greater and with the increase of tube flattening, the slope of the diagram becomes milder.

So, by considering the results that indicate the effect of this parameter on the pressure drop, the proper values for this parameter can be selected. Also, in the group of sensitive parameters, the tube flattening parameter has a high sensitivity for the pressure drop. As is shown in Figure 6b, with the increase in the inlet volumetric flow rate, the pressure drop also increases with a very sharp slope. So, the second most sensitive parameter is the inlet volumetric flow rate parameter. The other investigated parameter is the wall heat flux; and considering a near zero slope for the diagram showing the changes of the pressure drop versus wall heat flux (Fig. 6c), this parameter is not considered to be a sensitive parameter either for the pressure drop, and choosing different values for this parameter from its range of changes doesn't lead to a tangible change in the pressure drop values.

As Figure 6d demonstrates, the diagram showing the changes of the pressure drop versus the nanoparticle diameter selected, has very mild and near zero slopes. This indicates that by altering the values of this parameter in its respective ranges, no substantial change will be induced in the pressure drop.

The diagram of the pressure drop versus nanoparticle volume fraction has been shown in Figure 6e with a positive, and near zero, slope. With the change of nanoparticle volume fraction in its range of variations, a minor change is observed in the pressure drop, and so this parameter is not considered as a sensitive parameter for the pressure drop.

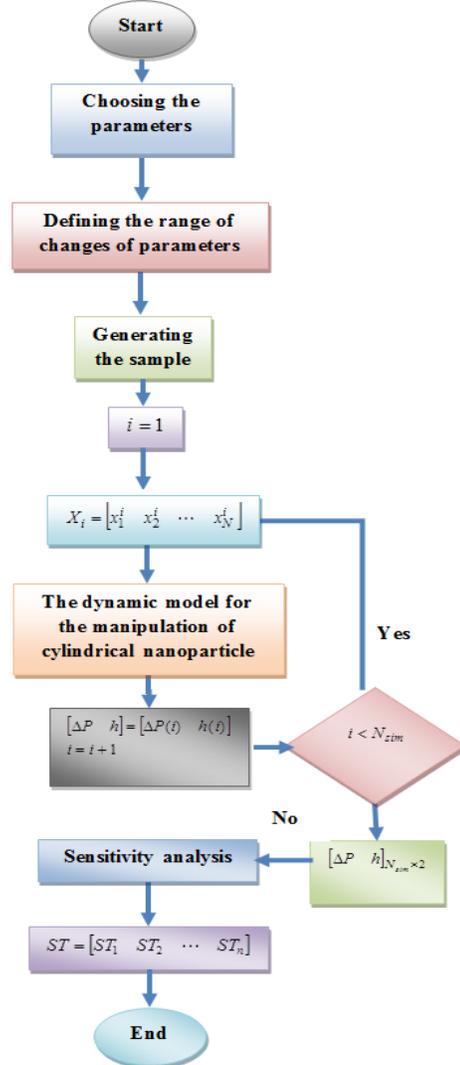


Fig. 5. Algorithm of sensitivity analysis

The changes of the heat transfer coefficient with tube flattening have been shown in Figure 7a.

According to this diagram, with the increase of tube flattening, heat transfer coefficient diminishes. As is observed in this figure, at low values of tube flattening, sensitivity is greater and with the increase of tube flattening, the slope of the diagram becomes milder. So, by considering the results that indicate the effect of this parameter on the heat transfer coefficient, the proper values for this parameter can be selected. Another sensitive parameter among the parameters is the inlet volumetric flow rate.

According to Figure 7b, with the increase of this parameter, the heat transfer coefficient increases with a linear slope. This linear increase indicates that the sensitivity of this parameter is the same in all its range of changes.

The diagram of the heat transfer coefficient versus wall heat flux has been shown in Figure 7c with a positive slope.

With the change of wall heat flux in its range of variations, a major change is observed in the heat transfer coefficient.

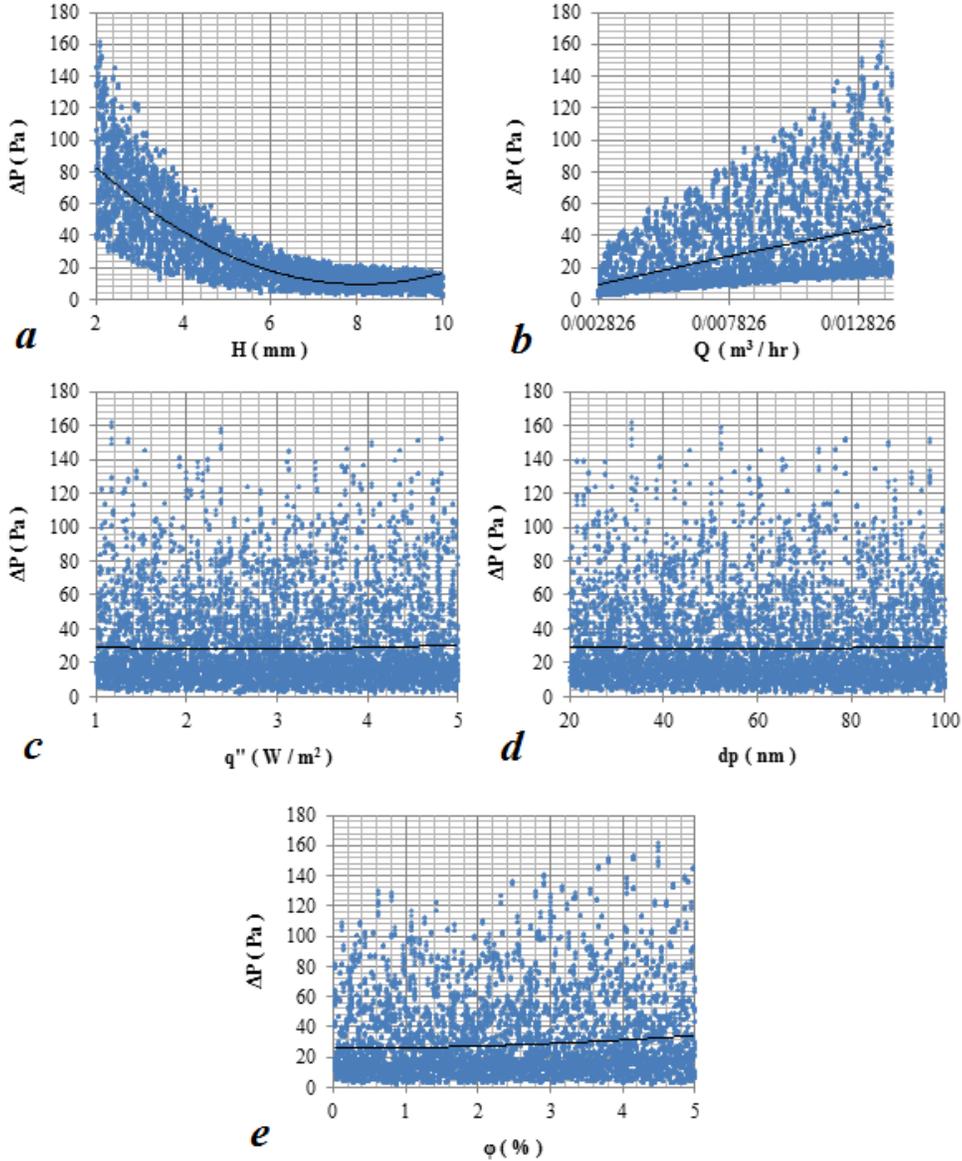
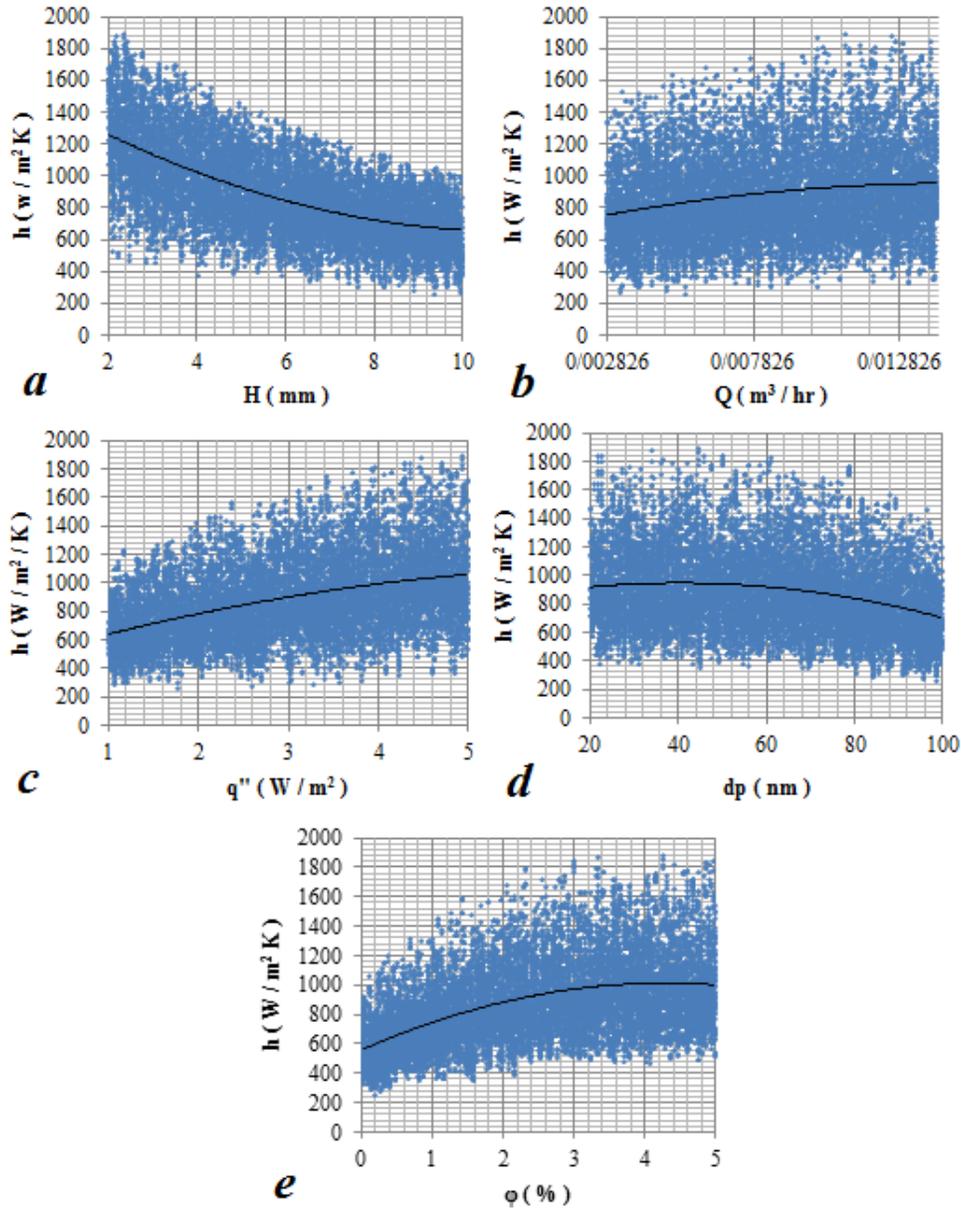


Fig. 6. The changes of pressure drop with: (a) tube flattening, (b) inlet volumetric flow rate, (c) wall heat flux, (d) nanoparticle diameter and (e) nanoparticle volume fraction

So, the third most sensitive parameter to which the heat transfer coefficient is sensitive is the wall heat flux. Another sensitive parameter among the input parameters is nanoparticle diameter.

According to Figure 7d, with the increase of this parameter, the heat transfer coefficient decreases. The second most sensitive parameter is the nanoparticle volume fraction. According to Figure 7e, with the increase of this parameter, heat transfer coefficient increases considerably.



**Fig. 7.** The changes of the heat transfer coefficient with: (a) tube flattening, (b) inlet volumetric flow rate, (c) wall heat flux, (d) nanoparticle diameter and (e) nanoparticle volume fraction

Figure 8 indicates more accurate analysis of the results obtained by the EFAST sensitivity analysis method. According to Figure 8, as expected, tube flattening (with a sensitivity index of 74%), inlet volumetric flow rate (with a sensitivity index of 24%) and nanoparticle volume fraction (with a sensitivity index of 2%), are of most significant sensitivity among five parameters in pressure drop. Also, According to Figure 8, tube flattening (with 40% sensitivity) is the most important parameter, and the parameters of inlet volumetric flow rate and wall heat flux (with 25% and 22% sensitivities, respectively) are the other effective parameters in heat transfer coefficient.

## CONCLUSION

In the present study, the effective parameters of water- $\text{Al}_2\text{O}_3$  nanofluid flowing in flat tubes were investigated using the EFAST Sensitivity Analysis (SA) method. The SA was performed using GMDH type artificial neural networks (ANN) which were based on validated numerical data of two phase modeling of nanofluid flow in flat tubes. There were five design variables namely: tube flattening (H), flow rate (Q), wall heat flux ( $q''$ ), nanoparticle diameter (dp) and nanoparticle volume fraction ( $\phi$ ) and there were two objective functions namely: pressure drop ( $\Delta P$ ) and heat transfer coefficient (h).

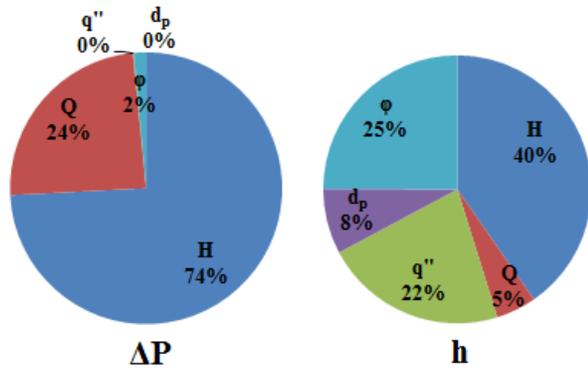


Fig. 8. Percent sensitivity of input parameter changes in the pressure drop and heat transfer coefficient

The results show that among design variables, the tube flattening has the highest effect on variations of pressure drop (74%) and heat transfer coefficient (40%). Except tube flattening, the flow rate and the nanoparticle volume fraction has the highest effect on pressure drop (24%) and heat transfer coefficient (25%) respectively. The effects of all of the design variables on objective functions were shown in the results (Fig. 8).

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