

MHD Boundary Layer Flow of a Nanofluid over an Exponentially Permeable Stretching Sheet with radiation and heat Source/Sink

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ABSTRACT: The problem of steady Magnetohydrodynamic boundary layer flow of an electrically conducting nanofluid due to an exponentially permeable stretching sheet with heat source/sink in presence of thermal radiation is numerically investigated. The effect of transverse Brownian motion and thermophoresis on heat transfer and nano particle volume fraction considered. The governing partial differential equations of mass, momentum, energy and nanoparticle volume fraction equations are reduced to ordinary differential equations by using suitable similarity transformation. These equations are solved numerically using an implicit finite difference scheme, for some values of flow parameters such as Magnetic parameter (M), Wall mass transfer parameter(S), Prandtl number(Pr), Lewis number (Le), Thermophoresis parameter (Nt), Brownian motion parameter(Nb), Radiation parameter (R). The numerical values presented graphically and analyzed for velocity, temperature and nanoparticle volume fraction.

KEYWORDS: Keller box; MHD; Nanofluid; Stretching permeable sheet; thermal radiation

INTRODUCTION

The analysis of boundary layer flow of viscous fluid and heat transfer due to stretching sheet has important application in Industry and engineering process and polymer industry, such as in polymer extrusion drawing of copper wires, artificial fibres, paper production, wire drawing, hot rolling, glass fibres, metal exclusion and metal spinning etc. Sakiadas [1] was probably the first to study the two dimensional boundary layer flow due to stretching wire in a fluid at rest. Crane[2] first studied the boundary layer flow due to linearly stretching sheet. The experimental result of TSOU et.al [3] confirmed that the mathematically described boundary layer problem on a continuous moving surface is physically reasonable. The flow and heat transfer over an exponentially stretching surface were investigated by Elbashbeshy [4] taking wall mass suction. Magyari and Keller [5] considered the boundary layer flow and heat transfer due to an exponentially stretching sheet.

Partha et al. [6] reported a similarity solution for mixed convection flow past an exponentially stretching surface. Ishak [7] studied the magneto hydrodynamic (MHD) boundary layer flow over an exponentially shrinking sheet in presence of thermal radiation. Bhattacharyya [8] discussed the boundary layer flow and heat transfer caused due to an exponentially shrinking sheet. N. Kishan and Kavitha [9] studied MHD Non-Newtonian power Law Fluid flow and Heat Transfer post a Non-Linear stretching

surface. Bhattacharyya and Pop [10] showed the effect of external magnetic field on the flow over an exponentially shrinking sheet.

The study of utilizing heat source or sink in moving fluids assume a great significance in all situations which deals with exothermic and endothermic reactions and those concerned with dissociating fluids. For physical situations, the average behaviour of heat generation or absorption can be expressed by some simple mathematical models because its exact modelling is quite difficult.

Heat source or sink has been assumed to be constant, space dependent. When technology processes take place at high temperatures thermal radiation heat transfer become very important and its effects cannot be neglected. Recent developments in hypersonic flights, missile re-entry rocket combustion chambers and gas cooled nuclear reactors, have focussed attention of researchers on thermal radiation as a mode of energy transfer and emphasize the need for inclusion of radiative transfer in these processes. For these many studies have appeared concerning the interaction of radiative flux with thermal convection flows. In certain porous media applications, working fluid heat generation (source) or absorption (sink) effects are important. Representative studies dealing with these effects have been reported by authors such that as Gupta and Sridhar [11], Abel and Veen [12] and Sharma [13]. In recent years, the flow analysis of Nanofluids has been topic of extensive research due to characteristic in increasing thermal conductivity heat transfer process.

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The term nanofluid was proposed by Choi[14] referring to the dispersion of nanoparticle in base fluid such as water, glycol, ethylene, propylene glycol. The nanofluid is an advance type of containing nano meter sized particle(diameter < 10 nm) are fibres suspended in the base fluid. Undoubtedly the nano fluids are advantages in the sense that they are more stable have an acceptable viscosity and better wetting, spready and dispersion properties on a solid surface. Nanofluids are used in different engineering applications such as microelectronics, micro fluidics, transportation, biomedical and manufacturing research on heat transfer in nanofluids has been receiving increased attention. Many researchers have found unexpected thermal properties of nanofluids and have proposed new mechanism behind the enhanced thermal properties of nanofluids. . The thermal conductivity enhancement characteristic of nanofluid was observed by Masuda et al. [15]. Buongiorno [16] discussed the reasons behind the enhancement in heat transfer for nanofluid and he found that Brownian diffusion and thermophoresis are the main causes. Later, Nield and Kuznetsov [17] and Kuznetsov and Nield [18] investigated the natural convective boundary layer flow of a nanofluid employing Buongiorno model. In 1827 the Scottish botanist Robert Brown observed that Microscopic pollen grains suspended in water move in a erratic, highly irregular, zig zag pattern. Following Brown's initial report other scientist verified the strange phenomenon. Brownian motion was apparent whenever very small particles were suspended in a fluid medium. For example smoke particles in air. It eventually determined that finer particles move more rapidly, that their motion is stimulated by heat, and that the movement is more active when the fluid viscosity is reduced. . It has a wide range of applications, including modeling noise in images, generating fractals, growth of crystals and stock market simulation. Thermophoresis is a phenomenon observed in mixtures of mobile particles where the different particle types exhibit different responses to the force of a temperature gradient. The term thermophoresis most often applies to aerosol mixtures, but may also commonly refer to the phenomenon in all phases of matter.

In recent years, it is found that thermophoresis phenomenon has many practical application in removing small particle gasstrings in determing exhaust gas particle trajectories from combustion devices, and in studying the particle material deposition on turbine Blades. Rosmila[19] et al. theoretically studied the problem of steady boundary layer flow of nanofluid past a porous stretching surface with variable stream conditions and chemical reaction. Rosca[20] et.al. have studied the steady forced convection stagnation point flow and mass transfer past a permeable stretching/shrinking sheet placed in a copper (cu) water based nanofluid. The boundary layer flow of nanofluid past a linearly stretching sheet was first studied by Khan and Pop[21] introducing the model of Nield and Kuznetsov[22]. The boundary layer flow induced in a nanofluid due to a

linearly stretching sheet with convective boundary condition was described by Makinde and Aziz [23]. Kandasamy et al. [24] investigated the MHD boundary layer flow of a nanofluid past a vertical stretching permeable surface with suction/injection. Rana and Bhargava [25] illustrated the steady, laminar boundary layer flow due to the nonlinear stretching of a flat surface in a nanofluid. Later, Hady et al. [26] analysed the boundary layer flow and heat transfer characteristics of a viscous nanofluid over a nonlinearly stretching sheet in the presence of thermal radiation and variable wall temperature. Hunegnaw and Naikoti Kishan[27] studied the MHD boundary layer flow and heat transfer of a nanofluid past a non-linearly permeable stretching/shrinking sheet with thermal radiation and suction effect in the presence of chemical reaction. N.Kishan et.al[28] studied MHD Boundary Layer Flow and Heat Transfer of a Nanofluid Over a Shrinking Sheet with Mass Suction and Chemical Reaction

Recently Krishnedu Bhattacharyya and G.C.Layek [29] studied the MHD boundary flow of nanofluid due to an exponentially permeable stretching sheet. Thus the motivation of this study is to examine the influence of thermal radiation and internal heat generation on magneto hydrodynamic boundary layer flow of nanofluid due to an exponentially stretching permeable sheet. The effects of Brownian motion and thermophoresis on heat transfer and nanoparticle volume fraction are considered. The governing equations are solved numerically using an implicit finite difference scheme known as Keller box method.

MATHEMATICAL FORMULATION

Consider the steady boundary layer flow of a nanofluid over an exponentially stretching sheet in presence of a transverse magnetic field. The governing equations of motion and the energy may be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2}{\rho_f} u \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{(\rho c)_p}{(\rho c)_f} \left[D_B \frac{\partial N}{\partial y} \frac{\partial T}{\partial y} + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \right] - \frac{1}{(\rho c)_p} \frac{\partial q_r}{\partial y} \quad (3)$$

$$u \frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} = D_B \frac{\partial^2 N}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2} \quad (4)$$

Where u and v are the velocity components in x - and y -directions, respectively, ν is the kinematic Viscosity, ρ_f is the density of the base fluid, T is the temperature, T_∞ is constant temperature of the fluid in the in viscid free stream, α is the thermal conductivity, $(\rho c)_p$ is the effective heat capacity of nanoparticles, $(\rho c)_f$ is heat capacity of the base fluid, N is nanoparticle volume fraction, D_B is the

Brownian diffusion coefficient, and D_T is the thermophoretic diffusion coefficient. Here, the variable magnetic field $B(x)$ is taken in the form

$$B(x) = B_0 \exp\left(\frac{x}{2L}\right), \quad \text{Where } B_0 \text{ is a constant.}$$

Using Rosseland approximation for radiation:

$$q_r = -\frac{4\sigma}{3k^*} \frac{\partial T^4}{\partial y'} \quad (5)$$

Where k^* is the mean absorption coefficient and σ is the Stefan Boltzmann constant. T^4 is expressed as a linear function of temperature by using Taylor series expansion about T_∞ is:

$$T^4 = 4T_\infty^3 T - 3T_\infty^4$$

The boundary conditions are given by

$$\begin{aligned} u &= U_w(x), \quad v = v_w \quad \text{at } y = 0, \quad u \rightarrow 0 \text{ as } y \rightarrow \infty, \\ T &= T_w = T_\infty + T_0 \exp\left(\frac{x}{2L}\right) \text{ at } y = 0, \quad T \rightarrow T_\infty \text{ as } y \rightarrow \infty, \\ N &= N_w = N_\infty + N_0 \exp\left(\frac{x}{2L}\right) \text{ at } y = 0, \quad N \rightarrow N_\infty \text{ as } y \rightarrow \infty, \end{aligned} \quad (6)$$

Where T_w is the variable temperature at the sheet with T_0 being a constant which measures the rate of temperature increase along the sheet, N_w is the variable wall nanoparticle volume fraction with N_0 being a constant and N_∞ is constant nanoparticle volume fraction in free stream. The stretching velocity U_w is given by

$$U_w(x) = c \exp\left(\frac{x}{L}\right), \quad (7)$$

here $c > 0$ is stretching constant. Here v_w is the variable wall mass transfer velocity and is given by

$$v_w(x) = v_0 \exp\left(\frac{x}{L}\right) \quad (8)$$

where v_0 is a constant with $v_0 < 0$ for mass suction and $v_0 > 0$ for mass injection.

Now, we introduce the similarity transformations:

$$\begin{aligned} \psi &= \sqrt{2\nu L c} f(\eta) \exp\left(\frac{x}{2L}\right), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \\ \varphi(\eta) &= \frac{N - N_\infty}{N_w - N_\infty}, \quad \eta = y \sqrt{\frac{c}{2\nu L}} \exp\left(\frac{x}{2L}\right) \end{aligned} \quad (9)$$

Where φ is the stream function with $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$ and η is the similarity variable

In view of relations in (6) we finally obtain the following self-similar equations:

$$f''' + ff'' - 2f'^2 - Mf' = 0 \quad (10)$$

$$\begin{aligned} (1 + \frac{4}{3}R)\theta'' + \text{Pr}(f\theta' - f'\theta + Nb\theta'\varphi' + \\ Nt\theta'^2) = 0 \end{aligned} \quad (11)$$

$$\varphi'' + \text{Le}(f\varphi' - f'\varphi) + \frac{Nt}{Nb}\theta'' = 0 \quad (12)$$

Where $M = 2\sigma B_0^2 L / c_\rho$ is the magnetic parameter, $\text{Pr} = \nu / \alpha$ is the Prandtl number and

$\text{Le} = \nu / D_B$ is the Lewis number, $R = \frac{4\sigma T_\infty^3}{kk^*}$ is the Radiation parameter. The dimensionless parameter Nb (Brownian motion parameter) and Nt (thermophoresis parameter) are defined as

$$\begin{aligned} Nb &= D_B \frac{(\rho c)_p}{(\rho c)_f} \frac{(N_w - N_\infty)}{\nu}, \quad Nt = \frac{D_T (\rho c)_p}{T_\infty (\rho c)_f} \frac{(T_w - T_\infty)}{\nu}, \\ R &= \frac{4\sigma T_\infty^3}{kk^*}, \end{aligned}$$

The boundary conditions (6) reduce to the following forms:

$$\begin{aligned} f(\eta) &= S, \quad f'(\eta) = 1 \quad \text{at } \eta = 0 \\ f'(\eta) &\rightarrow 0 \quad \text{as } \eta \rightarrow \infty, \\ \theta(\eta) &= 1 \quad \text{at } \eta = 0, \quad \theta(\eta) \rightarrow 0 \quad \text{as } \eta \rightarrow \infty, \\ \varphi(\eta) &= 1 \quad \text{at } \eta = 0, \quad \varphi(\eta) \rightarrow 0 \quad \text{as } \eta \rightarrow \infty, \end{aligned} \quad (13)$$

Where $S = -\nu_0 / \sqrt{\nu c / 2L}$ is the wall mass transfer parameter, $S > 0$ ($\nu_0 < 0$) corresponds to mass suction and $S < 0$ ($\nu_0 > 0$) corresponds to mass injection.

The quantities of physical interest for this problem are the local skin friction coefficient C_f , the local Nusselt number Nu_x , and the local Sherwood number Sh_x , which are respectively defined as

$$\begin{aligned} C_f &= \frac{\nu}{U_w^2 e^{2x/L}} \frac{\partial u}{\partial x} \Big|_{y=0}, \quad \sqrt{2Re_x} C_f = f''(0), \\ Nu_x &= -\frac{x}{[T_w - T_\infty] \partial Y} \Big|_{y=0}, \quad \frac{Nu_x}{\sqrt{2Re_x}} = -\sqrt{\frac{x}{2L}} \theta'(0), \\ Sh_x &= -\frac{x}{(N_w - N_\infty) \partial Y} \Big|_{y=0}, \quad \frac{Sh_x}{\sqrt{2Re_x}} = -\sqrt{\frac{x}{2L}} \varphi'(0) \end{aligned}$$

where $Re_x = U_w x / \nu$ is the local Reynolds number.

RESULTS AND DISCUSSION

The governing equations 10 to 12 under the boundary conditions (13) are solved numerically using the implicit finite difference scheme known as Keller Box method.

In order to validate our results, compared the values of skin friction coefficient $f''(0)$ in absence of magnetic field parameter and S wall mass transfer parameter. The results are found to be an excellent agreement comparison is shown in Table 1. The numerical computations have been performed for the velocity, temperature and nanoparticle volume fraction profiles for some values of

governing flow parameter such as Magnetic parameter (M), wall mass transfer parameter(S), Prandtl number(Pr), Lewis number (Le), thermophoresis parameter (Nt), Brownian motion parameter(Nb), radiation parameter (R). For different flow parameter the evolution of the similarity velocity profiles f' , temperature profile θ , and nanoparticle volume fraction ϕ are shown in Figures 1-7 respectively.

From Figure 1 it is seen that the influence of magnetic parameter is to reduce the velocity profiles f' whereas the magnetic parameter leads to enhance the temperature field.

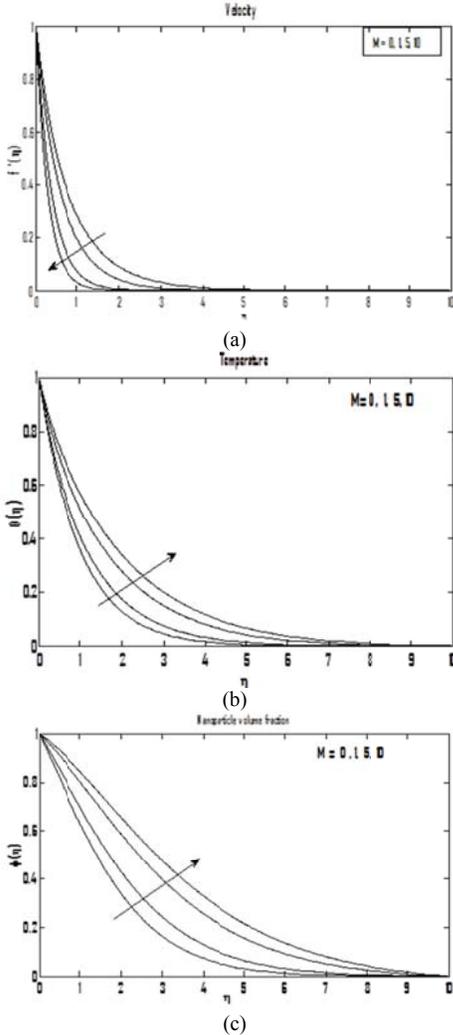


Fig. 1. Effects of Magnetic parameter on Velocity, Temperature and nanoparticle volume fraction

Table 1

The comparison of values of $f''(0)$ for $M = 0$ and $S = 0$.

	Magyari and Keller [29]	Present study
$f''(0)$	1.281808	1.28180838

This is due to the magnetic field, which opposes the transport process. Actually the increase of M leads to increase of Lorentz force arising because of interaction of magnetic and electric fields for the motion of an electrically

conducting fluid. The stronger Lorentz force produces much more resistance to the transport phenomena. From the other end, the temperature and the nanoparticle volume fraction increase with M .

The resistance offered to the flow is responsible in enhancing the temperature. Moreover, the Lorentz force has the tendency to increase the temperature and nanoparticle volume fraction in nanofluid motion.

In Figure 2, the velocity, temperature and nanoparticle volume fraction profiles are presented for selected values of heat source parameter.

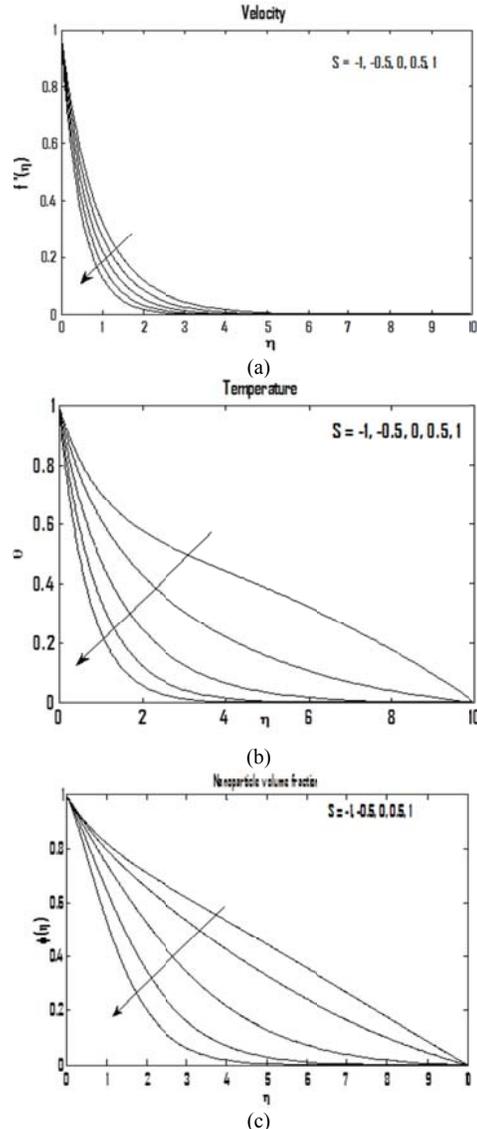


Fig. 2. Effects of Mass transfer parameter S on Velocity, Temperature and nanoparticle volume fraction

From the figure it is observed that the dimensionless velocity profile f' , temperature profile θ and nanoparticle volume fraction ϕ are decreases for increasing the strength of heat sink ($S > 0$) and due to increase of heat source strength ($S < 0$) the velocity, temperature and nanoparticle

volume fraction profile decreases. So, the thickness of momentum boundary layer and thermal boundary layer reduces for the increase of heat sink but it increases with heat source parameter.

This result is very significant for the flow where heat transfer is given prime importance.

The Figure 3 is represented the dimensionless temperature field and nanoparticle volume fraction for various values of Prandtl number Pr.

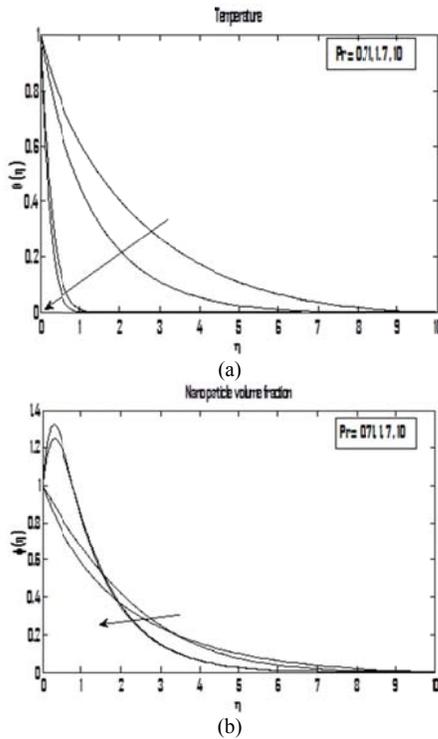


Fig. 3. Effects of Prandtl number Pr on Temperature and nanoparticle volume fraction

As Pr increases the dimensionless temperature as well as the thermal boundary layer thickness decreases quickly. The thermal boundary layer thickness reduces with Prandtl number it is due to decrease of thermal diffusivity for the larger Prandtl value number. The nanoparticle volume fraction overshoot near the boundary for higher values of Pr.

It is also notice that with increase of Prandtl number Pr leads to increases the nanoparticle volume fraction profile adjacent to the sheet and the opposite phenomenon is observed from the sheet. The influence of Lewis number the temperature and nanoparticle volume fraction profiles are depicted in Figure 4a and b respectively. The temperature profiles increase with increase of Lewis number Le and the effect is very meager. From Figure 4b it is evident that for the increasing the values of Lewis number Le, the nanoparticle volume fraction decreases significantly, and also the nanoparticle volume boundary layer thickness reduce sharply.

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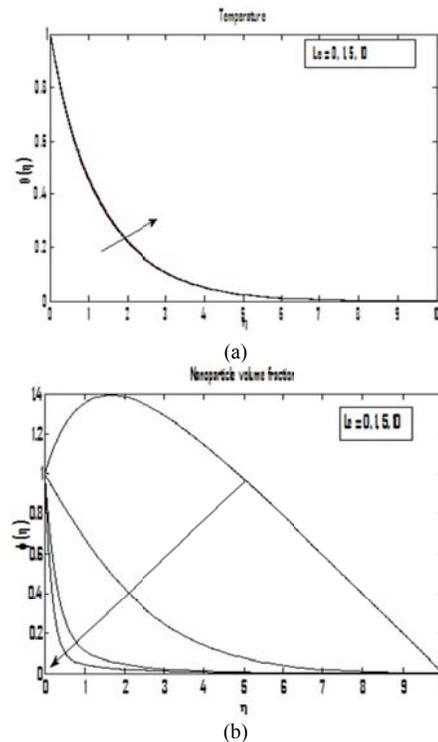


Fig. 4. Effects of Lewis number Le on Temperature and nanoparticle volume fraction

The impact of Brownian motion parameter Nb on the dimensionless temperature and nanoparticle volume fraction profiles are depict in Figure 5a and 5b respectively.

It can be seen from the figure that the temperature profile increases with the increase Brownian motion parameter Nb. The Figure 5b reveals that the nanoparticle volume fraction decreases with the increase of Brownian motion Nb. Also the nanoparticle volume of boundary layer thickness

decrease with the increase if Brownian motion parameter Nb.

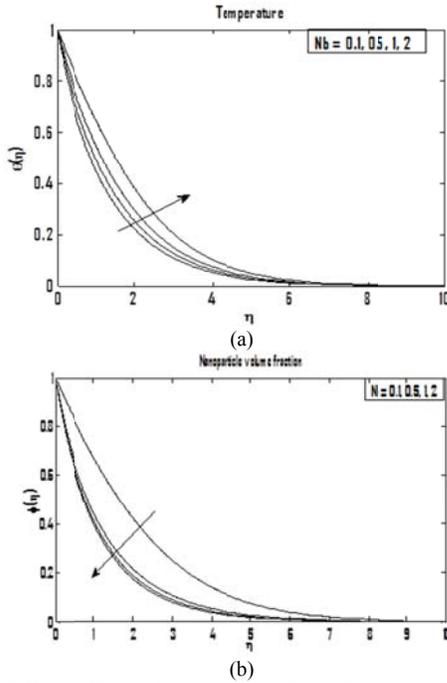


Fig. 5. Effects of Brownian parameter Nb on Temperature and nanoparticle volume fraction.

The nanofluid system due to the presence nanoparticle the Brownian motion takes place and the Brownian motion is affected with changes in Nb, and consequently the heat transfer characteristic of a fluid changes.

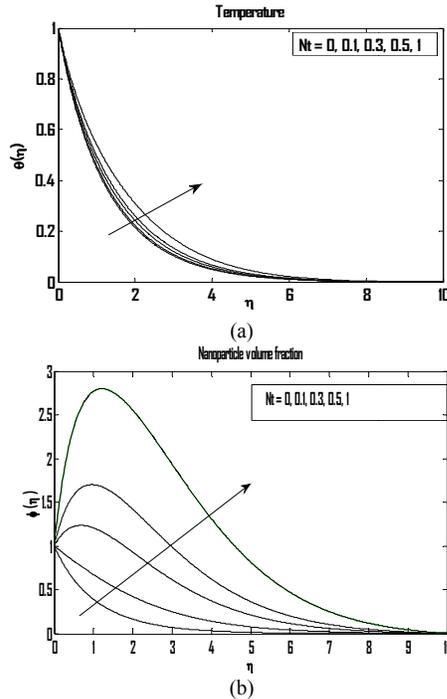


Fig. 6. Effects of thermophoresis parameter Nt on Temperature and nanoparticle volume fraction

Figure 6a and 6b shows the variation of dimensionless temperature and nanoparticle volume fraction with thermophoresis parameter Nt.

It is observed that the temperature profiles increases with increase of thermophoresis parameter Nt . Figure 6b reveals that the nanoparticle volume fraction increases significantly with the increase of Nt.

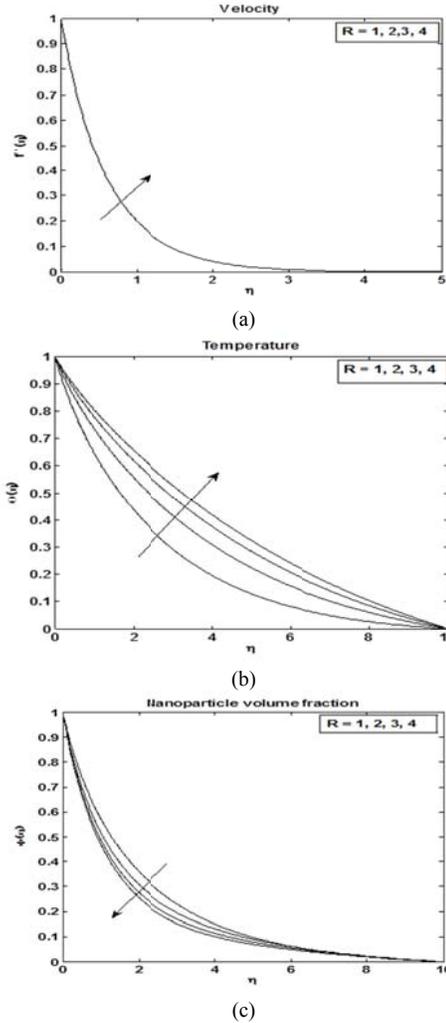


Fig. 7. Effects of Radiation parameter R on Temperature and nanoparticle volume fraction

It is also found the nanoparticle volume fraction overshoot near the wall, this is due to increase in Nt causes increment in the thermophoresis force which takes to move nanoparticle from hot to cold areas and consequently it increases the magnitude of temperature and nanoparticle volume fraction profiles. Interestingly, for slightly increase Nt leads to the large changes in the thickness of nanoparticle boundary layer.

Here from 7(a-c) demonstrates the variation of temperature profiles for the various values of thermal radiation parameter R, respectively for velocity, temperature and nanoparticle volume fraction from the

plots it is evident that the effect of thermal radiation is to analyze the velocity and temperature profiles while the nanoparticle volume fraction reduces with the increase of radiation.

CONCLUSION

The present paper is to analyze the laminar MHD boundary layer flow of nanofluid past and exponentially permeable stretching sheet. The main focus is to predict the effects of Brownian motion, thermophoresis, and radiation on heat of mass transfer in boundary layer flow of nanofluid. The finding of the present study is conclude as.,

1. Due to the increase of magnetic field the velocity profile decreases and the temperature and nanoparticle volume fraction reduces.

2. With the increase of velocity, temperature and nanoparticle volume fraction decreases the heat Source whereas increases with heat sink.

3. For Brownian motion the temperature of the fluid increases and the nanoparticle volume fraction decreases the effect of thermophoresis parameter is to enhance the temperature profiles and nanoparticle volume fraction significantly.

4. With the increase of Prandtl number and Lewis number the nanoparticle volume fraction decreases, the temperature profile decreases with Prandtl number and decreases with Lewis number.

5. The effect of radiation parameter is to enhance the velocity and temperature and reduces the nanoparticle volume fraction.

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