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ORIGINAL RESEARCH PAPER

Single Walled Carbon Nanotube Effects on Mixed Convection heat Transfer in an Enclosure: a LBM Approach

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Abstract

The effects of Single Walled Carbon Nanotube (SWCNT) on mixed convection in a cavity are investigated numerically. The problem is studied for different Richardson numbers (0.1-10), volume fractions of nanotubes (0-1%), and aspect ratio of the cavity (0.5-2.5) when the Grashof number is equal to 103. The volume fraction of added nanotubes to Water as base fluid are lowers than 1% to make dilute suspensions. The Study presents a numerical treatment based on LBM to model convection heat transfer of Carbon nanotube based nanofluids. A theoretical model is used for effective thermal conductivity of the nanofluid containing Carbon nanotubes. This model covers different phenomena of energy transport in nanofluids. Also, an analytical model is applied for effective viscosity of the nanofluid which includes the Brownian effect and other physical properties of nanofluids. Results show that adding a low value of SWCNT to the base fluid led to significant enhancement of convection heat transfer. Make a comparison between the obtained results and other published papers shows that Carbon nanotubes enhances the rate of convection heat transfer better than other nanoparticles.

1. Introduction

The enhancement of fluids heat transfer is a very interesting topic for different kinds of industrial and engineering applications. A well-known way to enhance the rate of convection heat transfer of conventional fluids such as water, oil and ethylene glycol which has low thermal conductivity [1,2] is adding nano scale conductive particles. The added particles can be metals [3], non metals [4] or Carbon nanotubes [5, 6]. Because of the high thermal conductivity of these particles, they can improve conductivity of the suspension systematically. .Nowadays, nanoparticle added fluids are known as nanofluid that first time Choi [7] named this kind of fluid suspensions. He presented enhancement of convection heat transfer by adding nanoparticles to the fluids.

In recent years, many studies conducted to study heat transfer of nanofluids numerically and experimentally [8-14]. Khanafer et al. [8] presented a heat transfer enhancement by adding nanoparticles to fluid in a two dimensional enclosures at natural

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Nomenclature

c	Discrete lattice velocity
c_s	Speed of sound in Lattice scale
c_p	Specific heat at constant pressure $(kj.kg^{-1}.k^{-1})$
F	External force in direction of lattice velocity
f , g	Distribution function
g_y	Acceleration due to gravity $(m.s^{-2})$
Gr	Grashof number $(g\beta(T_h - T_c)L^3/\nu^2)$
H	Height of Cavity (m)
k	Thermal conductivity $(W.m^{-1}.k^{-1})$
Ma	Mach number
Nu	Nusselt number
Pr	Prandtl number (ν/α)
Re	Reynolds number $(u_0.W/\nu)$
Ri	Richardson number (Gr/Re^2)
T	Dimensional temperature (K)
u_0	Velocity of lid $(m.s^{-1})$
W	Width of cavity (m)

Greek symbols

ψ	Dimensionles sstream-function $(\psi = \overline{\psi}/Wu_0)$
X, Y	Dimensionless coordinate
α	Discrete lattice direction
β	Thermal expansion coefficient $(1.k^{-1})$
θ	Dimensionless temperature
Δt	Lattice time step
ρ	Density $(kg.m^{-3})$
τ	Lattice relaxation time
φ	Solid volume fraction
w	Weighting factor

Subscripts

ave	Average
c	Cold
eq	Equilibrium
f	Fluid
h	Hot
nf	Nanofluid
s	Solid particle
(-)	Dimensional Parameters

convection regime for the different Grashof numbers. They presented an increase of Nu_{ave} about 7.5 %, 12%, 15.5% and 20%, respectively, by adding a volume fraction of added Cu nanoparticles to the Water equal to 4%, 6%, 8% and 10% when $Gr = 10^4$. Also, this increase values are about 7%, 10%, 15.5% and 19% for $Gr = 10^5$ at same volume fractions. Gherasim et al. [9] conducted to an experimental study and represented heat transfer enhancement possibility of coolants with suspended Al_2O_3 nanoparticles dispersed in water inside a radial flow cooling device. Saleh et al. [10] studied convection heat transfer of a nanofluid-filled trapezoidal enclosure. They investigated the effect of different volume fraction of Al_2O_3 and Cu nanoparticles on heat transfer enhancement of Water as base fluid at different Grashof numbers. They reported an enhancement of natural convection heat transfer about 2.4%, 4.7%, 7.1%, 9.5% and 11.9% by adding a $\varphi = 1,2,3,4$ and 5% of Al_2O_3 nanoparticles when the angle of side walls are 23.96° at $Gr = 10^5$ and also for Cu nanoparticles at same condition the values are about 3.9%, 7.9%, 11.8%, 15.7%, 19.5%.

In 1991, Iijima [15] discovered Carbon nanotubes as an allotrope of Carbon which is made of long-chained molecules of Carbon with Carbon atoms arranged in a hexagonal complex to form a tubular structure. Single Walled (SWCNT), Double Walled (DWCNT) and Multi Walled (MWCNT) are different classes of Carbon nanotubes depending on the number of tubules of Carbon in their structure.

In last decade Carbon nanotubes has been mentioned as attractive topic by many researches which is generally due to their special properties at physical view such mechanical and thermal properties. At this point of view, the Carbon nanotubes have extraordinary thermal properties such as thermal conductivity about twice as high as diamond [16] or thermal stability up to 2800 C in vacuum. The higher thermal conductivity of Carbon nanotubes relative to other nanoparticles led to that the nanofluids containing cylindrical Carbon nanotubes are expected

to have better heat transfer properties compared with the other nanofluids with spherical nanoparticles [17, 18]. Gavili et al. [19] simulated the mixed convection in a two-sided lid-driven differentially heated square cavity for nanofluids containing Carbon nanotubes for nanofluids. They used the new theoretical model for thermal conductivity of the Carbon nanotubes, but the effective viscosity was calculated with the Maxwell model.

The progress of using the Lattice Boltzmann Method (LBM) as a numerical technique to simulate the heat transfer and fluid flow has been obvious in the last decade [20-27]. The Lattice Boltzmann Method has well-known advantages such easy implementation, possibility of parallel coding and simulating of complex geometries [21, 22], fuel cell [23], porous media [24], nanofluids [25-27] and etc. Fattahi et al. [25] applied Lattice Boltzmann Method to study natural convection heat transfer of Al_2O_3 and Cu nanofluid in a cavity. Their results show that increasing the solid volume fraction led to a heat transfer enhancement at any Rayleigh number and also heat transfer increases with increase in Rayleigh number for a particular volume fraction.

The mixed convection heat transfer of different nanoparticle-based nanofluid in a lid-driven cavity was be mentioned in current years [27-29]. Nemati et al. [27] applied LBM to investigate the effect of Cu, Al_2O_3 and CuO nanoparticles on mixed convection in a lid-driven cavity. Their results show that adding nanoparticles increase the rate of mixed convection heat transfer of the base fluid for all tested Reynolds numbers. Abu-Nada and Chamkha [28] conducted to study of mixed convection heat transfer of Water-Al_2O_3 nanofluid in an inclined cavity. Heat transfer enhancement due to increase of nanoparticles volume fraction at different Richardson and Grashof numbers was presented in their results. Their results show that Nu_{avg}on hot wall enhances about 3.3%, 8.4%, 11.9% and 17.2% respectively for $\varphi = 2,5,7$and 10% at $R_i = 0.2$ and $Gr = 10^5$ when the inclination angle of the cavity is equal to zero and also this values are 3%, 8.5%, 12 % and 17% for $R_i = 2$.

The main aim of the present study is to investigate the heat transfer enhancement of the Carbon nanotubes in the lid-driven cavity. In this simulation the effective conductivity and viscosity were calculated based on the new theoretical models. The Sabbaghzadeh and Ebrahimi [30] and Masoumi et al. [31] models are applied for effective thermal conductivity and effective viscosity of the nanofluid, respectively. The new thermal conductivity model captures the effects of different phenomena of energy transport in nanofluids. At molecular point of view, some of the most important incidents in energy transport are: Collision among the base fluid molecules, thermal diffusion of nanoparticles, thermal diffusion in the nanolayer in the fluid, thermal interaction of dynamic complex nanoparticles with the base fluid molecules and finally collision between nanoparticles caused by Brownian motion.

By considering the extraordinary thermal conductivity and low density of carbon nanotubes at comparison with other nanoparticles and also using a good expansion of effective thermal conductivity of nanofluids which covers molecular phenomena of energy transport of nanofluids, it is predicted that a great heat transfer enhancement can be achieved in this study.

To the best of author knowledge, a lack of using Lattice Boltzmann method (LBM) as a suitable numerical method to study of convection heat transfer of nanofluids containing Carbon nanotubes is sensible. Therefore, in this study the used numerical method is LBM with coupled double population approach for flow and temperature fields. Also the Bounce Back method is applied in LBM to model boundary condition in solid boundaries which supply second order accuracy in both flow and temperature fields. The effect of the volume fraction of the SWCNT-particles on the local Nusselt number distribution, average Nusselt number, streamlines and temperature contours is investigated for various Richardson numbers when Grashof number is fixed to . Also the aspect ratio of the cavity is studied as an important geometric parameter of the 2D cavities in different condition to exhibit the role of this parameter on both heat transfer rate and fluid flow of the base fluid and the nanofluid.

2. The Numerical Procedure

2.1. Lattice Boltzmann Method for Flow and Thermal Fields

The LBM utilizes two distribution functions, for the flow and temperature fields. It uses modeling of movement of fluid particles to define macroscopic parameters of fluid flow. The basic form of LBM applies uniform Cartesian cells to discrete problem domain. Each cell of the grid has a constant number of distribution functions, which represent the number of fluid particles movement in these separated

directions. The distribution functions are obtained by solving the lattice Boltzmann Equation (LBE), which is a special form of the Kinetic Boltzmann Equation.

The basic form of the Lattice Boltzmann Equation with an external force by introducing BGK approximations can be written as follows for the both flow and the temperature fields [20]:

$$f_\alpha(x+e_\alpha\Delta t, t+\Delta t) = f_\alpha(x,t) + \frac{\Delta t}{\tau_m}\left[f_\alpha^{eq}(x,t) - f_\alpha(x,t)\right] + \Delta t e_\alpha \quad (1)$$

$$g_\alpha(x+e_\alpha\Delta t, t+\Delta t) = g_\alpha(x,t) + \frac{\Delta t}{\tau_t}\left[g_\alpha^{eq}(x,t) - g_\alpha(x,t)\right] \quad (2)$$

Where $f_\alpha(x,t)$ is a distribution function on the mesoscopic level. Here $c = \frac{\Delta x}{\Delta t}$ is a streaming speed where Δx and Δt are the lattice cell size and the lattice time step size, respectively. While e_α is the discrete lattice velocity in direction α and f_α is the external force term in the direction of discrete velocity. Also, w_α is a weighting factor and ρ is the lattice fluid density. τ_m and τ_t are the dimensionless collision-relaxation times for the flow and temperature fields, respectively. They defined as follows [32]:

$$\tau_m = 0.5 + \frac{Ma.W\sqrt{3\,\mathrm{Pr}}}{c^2\Delta t\sqrt{Ra}}$$
$$\tau_t = 0.5 + \frac{3v}{\mathrm{Pr}\,c^2\Delta t} \quad (3)$$

Characteristic velocity for both natural ($V_{natural} = \sqrt{\beta g_y \Delta T H}$) and force ($V_{force} = vRe/W$) regimes must be small in comparison with the fluid speed of sound. By considering D2Q9 model (Fig.1) for applied lattice scheme for both flow and temperature fields, equilibrium distribution functions for flow field (f_α^{eq}) and temperature field (g_α^{eq}) are calculated as follows in different α directions:

$$f_\alpha^{eq} = w_\alpha\rho[1 + 3(e_\alpha.\vec{u}) + \frac{9}{2}(e_\alpha.\vec{u})^2 - \frac{3}{2}\vec{u}^2] \quad (4)$$

$$g_0^{eq} = -w_0\rho RT[\frac{3}{2}\vec{u}^2]$$
for $\alpha = 1-4$:
$$g_\alpha^{eq} = w_\alpha\rho RT[\frac{3}{2} + \frac{3}{2}(e_\alpha.\vec{u}) + \frac{9}{4}(e_\alpha.\vec{u})^2 - \frac{3}{2}\vec{u}] \quad (5)$$

for $\alpha = 5-8$:
$$g_\alpha^{eq} = w_\alpha\rho RT[3 + 6(e_\alpha.\vec{u}) + \frac{9}{2}(e_\alpha.\vec{u})^2 - \frac{3}{2}\vec{u}^2] \quad (6)$$

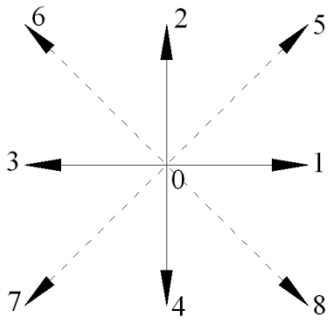


Fig.1. Discrete velocity vectors for the D2Q9 model of LBM.

The D2Q9 lattice is a Cartesian 2D lattice with nine velocity directions. For these model the values of e_α and w_α for various α directions will be, respectively:

$$e_\alpha = \begin{cases} (0,0) \\ c(\cos\theta_\alpha + \sin\theta_\alpha) & , \ \theta_\alpha = (\alpha-1)\pi/2 \\ \sqrt{2}c(\cos\theta_\alpha + \sin\theta_\alpha) & , \ \theta_\alpha = (\alpha-5)\pi/2 + \pi/4 \end{cases} \quad (6)$$
$for\ (0,0): \alpha = 0\ , forc(\cos\theta_\alpha + \sin\theta_\alpha): \alpha = 1,2,3,4, for\sqrt{2}c(\cos\theta_\alpha + \sin\theta_\alpha): \alpha = 5,6,7,8$

$$w_\alpha = \begin{cases} 4/9 & \alpha = 0 \\ 1/9 & \alpha = 1,2,3,4 \\ 1/36 & \alpha = 5,6,7,8 \end{cases} \quad (7)$$

In order to incorporate external force in collision part of Lattice Boltzmann model (Eq.1), radiation heat transfer and viscous dissipation are neglected at the numerical simulation. Therefore, to capture buoyancy force effects in the flow field, the Boussinesq approximation is applied. Thus, to model buoyancy force in Eq. (1), the external force needs to be assumed as below in the needed direction:

$$F_\alpha = 3w_\alpha g_\alpha \beta (T - T_{ref}) \quad (8)$$

Finally, macroscopic variables can be calculated as follows:

$$\begin{aligned} &\text{Flow density:} && \rho = \sum_\alpha f_\alpha \\ &\text{Momentum:} && \rho u_i = \sum_\alpha f_\alpha c_{i\alpha} \\ &\text{Temperature:} && T = \sum_\alpha g_\alpha \end{aligned} \quad (9)$$

4.1. Nanofluids Modeling

At present study, nanofluid is assumed as a single phase fluid. The added nanoparticles to the base fluid changes thermal conductivity, viscosity and other basic characteristic of the base fluid, thus innovated Carbon-based nanofluid has special characteristic as a combination of the pure fluid and Carbon nanotubes. Thermal diffusivity of nanofluid is as follows:

$$\alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}} \quad (10)$$

The density, heat capacitance and thermal expansion of nanofluid can be defined as [33]:

$$\begin{aligned} \rho_{nf} &= (1-\varphi)\rho_f + \varphi\rho_s \\ (\rho c_p)_{nf} &= (1-\varphi)(\rho c_p)_f + \varphi(\rho c_p)_s \\ \beta_{nf} &= (1-\varphi)\beta_f + \varphi\beta_s \end{aligned} \quad (11)$$

2.2.1. Effective Viscosity of Nanofluid

The effects of nanoparticles on the viscosity of nanofluids are introduced by the so-called apparent viscosity which is presented by μ_{app} . The numerical procedure to simulate the nanofluid viscosity is same as those presented by Masoumi et al. [31]. According to this analytical model of nanofluid viscosity, the effective viscosity can be defined as follows:

$$\mu_{nf} = \mu_{bf} + \mu_{app} = \mu_{bf} + \frac{\rho_{np} V_{Brownian} d_{np}}{72 C \delta} \quad (12)$$

As presented, Masoumi et al. considered Brownian motion as an important parameter that creates a relative velocity between the nanoparticle and the base fluid in nanofluids [31]. V_B is the Brownian velocity and the corresponding Reynolds number are defined as [34]

$$V_{Brownian} = \frac{1}{d_{np}} \sqrt{\frac{18 K_b T}{\pi \rho_{np} d_{np}}} \quad (13)$$

$$Re_{Brownian} = \frac{1}{v_{bf}} \sqrt{\frac{18 K_b T}{\pi \rho_{np} d_{np}}} \quad (14)$$

While, k_b is the Boltzmann Constant ($=1.38\times10^{-23}\ JK^{-1}$). Also,$\delta$ is the approximately distance between the centers of particles and it will be obtained from [31]:

$$\delta = \sqrt[3]{\frac{\pi}{6\varphi}} d_{np} \quad (15)$$

Now the correction factor, C, must be determined. Using the experimental data associated with the nanofluid viscosity c_1, c_2, c_3 and c_4 can be determined.

$$C = \mu_{bf}^{-1}[(c_1 d_{np} + c_2)\varphi + (c_3 d_{np} + c_4)] \quad (16)$$

In this study the contant variables was calculated based on the experimental data of Yang Hong and Choi [35].

2.2.2. Thermal Conductivity of Nanofluid

The applied effective thermal conductivity of Carbon nanotubes-based nanofluid is same as those proposed by Sabbaghzadeh and Ebrahimi [30]. This Theoretical model contains five phenomena of energy transport in nanofluids: collision among the base fluid molecules; thermal diffusion of nanoparticles, thermal diffusion in the nanolayer in the fluid; thermal interaction of dynamic complex nanoparticles (nanoparticles and nanolayer) with the base fluid molecules and finally collision between nanoparticles due to Brownian motion. Their theoretical model can be expressed as [30]:

$$\begin{aligned} K_{eff} &= K_{bf}[1-\varphi M^2] + \varphi(\kappa K_{np} + K_{nl}(M^2 - 1)) \\ &+ \varphi K_f M^2 \frac{df}{D}\left(0.35 + 0.56 Re_{bf}^{0.52}\right) pr_{bf}^{0.3} \end{aligned} \quad (17)$$

Where K, D and M are respectively thermal Kapitza resistance, diameter of nanoparticles complex and ratio of radius complex nanoparticle to nanoparticle which is defined as follows:

$$M = \frac{r_{nl} + r_{np}}{r_{np}} \tag{18}$$

The term of K_{nl} in Eq.17 indicates the thermal conductivity of nanolayer which has a value equal to:

$$K_{nl} = \frac{K_{bf} r_{np} (M \frac{K_{np}}{K_{bf}} - 1) \ln(M)}{d_{np} \ln(M \frac{K_{np}}{K_{bf}})} \tag{19}$$

The relation is valid for $10^{-1} < \mathrm{Re}_{bf} < 10^{5}$, The Reynolds number, Re_{bf} is defined as follows:

$$\mathrm{Re}_{bf} = \frac{\overline{u} L}{v_{bf}} \tag{20}$$

u^* is the mean velocity of the complex nanoparticles and L is the specific length of the cylindrical shape that can be calculated as follows:

$$L = \sqrt[3]{\frac{\pi r_{np}^2 l M^2}{\varphi}} \tag{21}$$

Where, l is the cylinder length which assumed to have a value equal to 10 micrometer ($l = 10 \mu m$). From the molecular theory of heat, $\overline{u}$ is determined as follows [36]:

$$\overline{u} = \sqrt{\frac{3 K_b T}{m_n}} \tag{22}$$

As cited, K_b is the Boltzmann Constant, m_n is the mass of the complex nanoparticle which can be calculated as follows:

$$m_n = \pi r_{np}^2 l \rho_{np} \left[\frac{\rho_{bf}}{\rho_{np}} (M^2 - 1) + 1 \right] \tag{23}$$

Sabbaghzadeh and Ebrahimi [30] presented more details of thermal conductivity calculation in the nanofluids based on carbon nanotube existence in convective fluids in their published research paper.

3. Main Problem

3.1. Problem Description

In the present study, the effects of single walled Carbon nanotubes on mixed convection heat transfer in a 2D deep lid-driven cavity are investigated using the Lattice Boltzmann Method based on double population approach and bounce back method. The vertical walls of the cavity are assumed to be insulated while the bottom wall is maintained at a uniform temperature (T_h) differentially higher than the cold lid-wall temperature (T_c). The lid wall has a constant velocity motion (u_0) in a left to right direction equal to about 0.1 times of the sound speed (Fig.2) to satisfy that the code works near incompressible regime by considering acceptable Mach number($Ma = \frac{u_0}{c_s} < 0.1$) in Lattice Boltzmann Method. The effect of the volume fraction of single-walled Carbon nanotubes on both fluid flow and heat transfer is investigated for different Richardson numbers at the range of $0.1 \le Ri \le 10$, for $\mathrm{Gr} = 10^3$, while the Prandtl number of based fluid (Water) at reference temperature ($22^{\circ}C$) is 6.57.

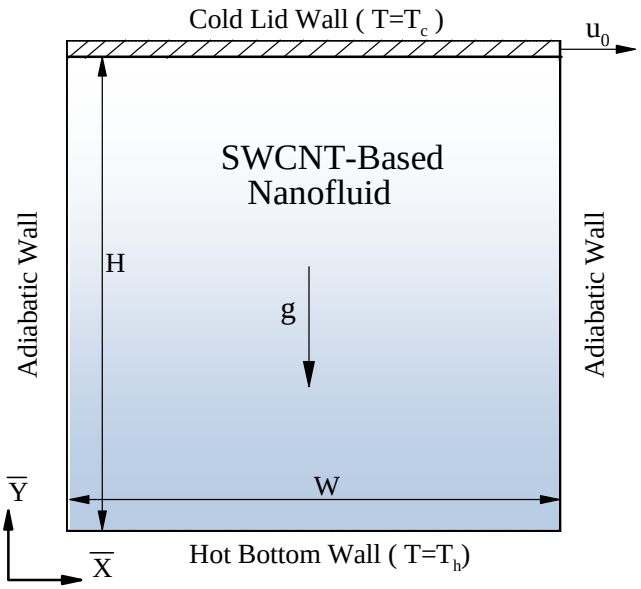


Fig.2. Schematic geometry of the problem.

It is assumed that the nanotubes are in constant form with uniform diameter which are at thermal equilibrium state with base fluid and also the nanotubes have same velocity with the base fluid thus there is no sleep velocity between added nanotubes and molecules of the base fluid. Thermo-physical properties of SWCNT-particles and Water at reference temperature are tabulated in Table1. The dimensionless quantities are as follows:

$$u = \frac{\bar{u}}{u_0} \tag{24}$$

$$Re = \frac{\rho u_0 W}{\mu}$$

Table .1
Thermo-physical properties of the base fluid and nanoparticles [19].

Property	Water	SWCNT
$c_p(J/KgK)$	4179	709.0
$\rho(Kg/m^3)$	997.1	2200
$k(W/mK)$	0.613	2000
$\beta\times10^5(K^{-1})$	21	1.5
$d_p(nm)$	0.384	10

The boundary conditions on the solid walls are in the following forms:

$$\begin{cases} u=v=0, \ \theta=1, & for\ y=0\ ,\ 0\le x\le 1 \\ u=v=0, \ \theta=0, & for\ y=A\ ,\ 0\le x\le 1 \\ u=v=0, \ \dfrac{\partial\theta}{\partial x}=0, & for\ 0<y<A\ ,\ x=0 \\ u=v=0, \ \dfrac{\partial\theta}{\partial x}=0, & for\ 0<y<A\ ,\ x=1 \end{cases} \tag{25}$$

The local Nusselt number is defined along the hot bottom wall as follows:

$$Nu = \left| W\left(\frac{k_{nf}}{k_f}\right)\frac{\partial\theta}{\partial y}\right| \tag{26}$$

Where, W is the width of the cavity. By integrating the local Nusselt number along the X direction, the average Nusselt number is calculated as follows:

$$Nu_{ave} = \frac{1}{L}\int_0^L Nu dx \tag{27}$$

3.2. Code Validation, Solution Convergence and Mesh Independent Treatment

The numerical code is validated by the results presented by Moallemi and Jang [37] for mixed convection in a lid-driven cavity at $Re = 500$, $Ri = 0.4$, $Pr=1$ and $A=1$. (Fig.3) also for more consistency, the Nusselt number distribution along the cavity lid is compared with result of Moallemi and Jang [37]. The agreement of the result with the pervious results confirms the accuracy of numerical code to simulate the flow and temperature fields of convection heat transfer.

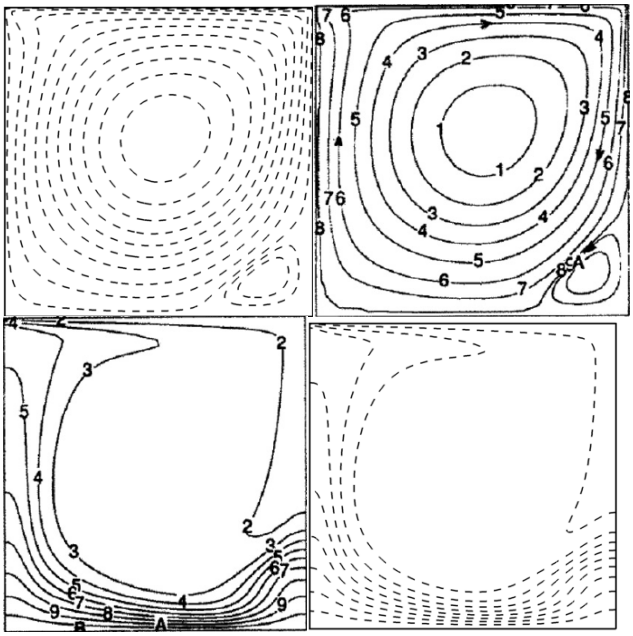


Fig.3. Streamline and Temperature contours for R_e= 500 , R_i = 0.4 and P_r = 1.Top: Moallemi and Jang[37] bottom: Present study.

A grid independency treatment is used to select an adequate number of nodes in the mesh grid. Figure 5 shows the values of Temperature at vertical midsection of the cavity and also u and v component of the velocity at horizontal mid-section of the cavity

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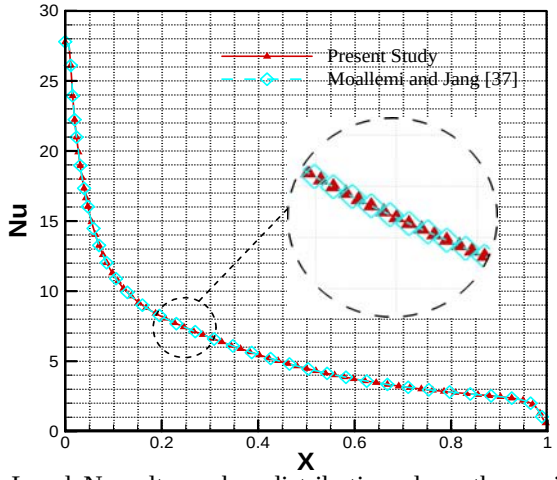


Fig.4. Local Nusselt number distribution along the cavity lid for $R_e = 500$, $R_i = 0.4$ and $Pr = 1$.

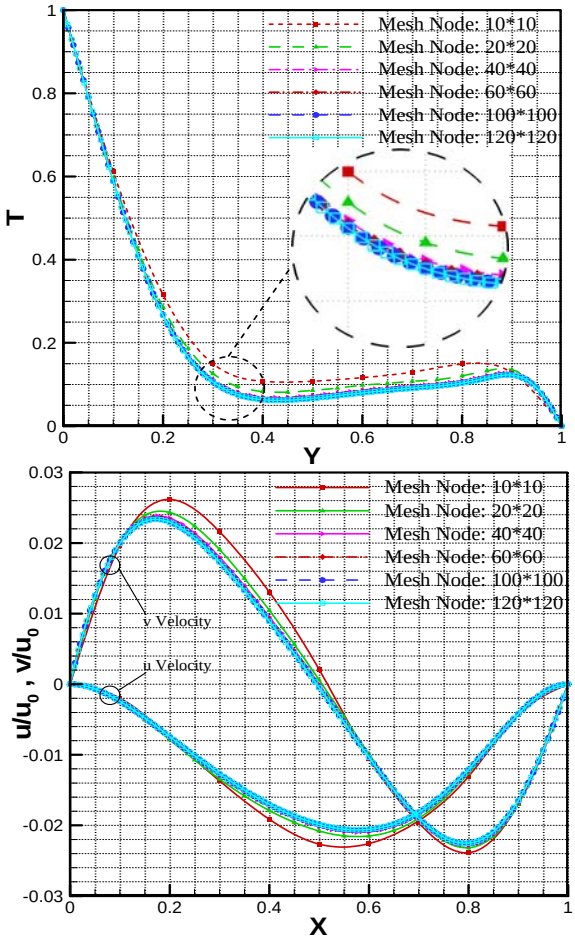


Fig.5. Mesh independent results of the cavity at $Ri = 1$, $\varphi = 0.25\%$ and $A = 1$ .a) Temperature in vertical mid-section b) Velocity components in horizontal mid-section

This treatment shows that a grid with100 nodes in each direction can satisfy the grid independency treatment when aspect ratio of the cavity is equal to unity.lattice nodes in X direction is used and number of nodes in Y direction is 50 to 250 for aspect ratio of 0.5 to 2.5 , respectively.To get a steady state result in the numerical solution, the results are compared at different time steps. Figure.6 shows the local Nusselt number along both top and bottom walls for various numbers of time steps. As indicated, when the number of time steps is more than 7.5×10^4 the results converge to a steadystate and increasing of the time step after this condition has no clear effect on the values of needed parameters. This algorithm was done for all case studies to get convergent results by applying an error function as presented in the following equation in the used numerical code:

$$Err = \frac{\left(\Psi^{n}_{i,j} - \Psi^{n-1}_{i,j}\right)}{number\ of\ mesh\ nodes} \le 10^{-5} \qquad (28)$$

Where, $\psi^{n}_{i,j}$ stands for a variable of flow or temperature fields such as temperature or velocity component in each node of the grid at n_{th} iteration. Steady state solution was declared when the relative change in the variables of flow and temperature fields between two consecutive iterations at through solution domain is satisfied by the presented criterion.

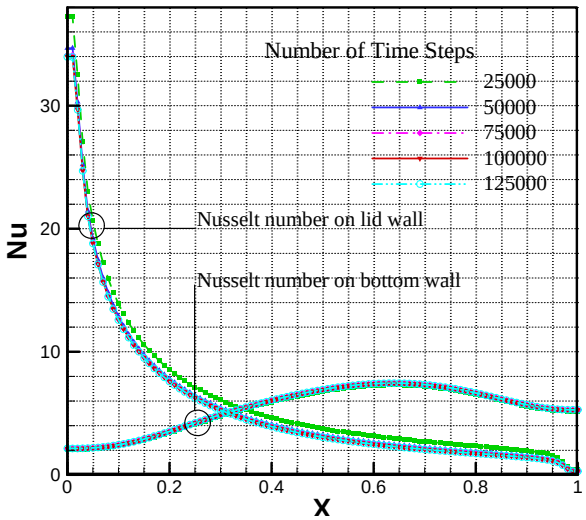


Fig.6. Local Nusselt number distribution for different time steps at $Ri = 0.1$, φ=0.5% and $A = 1$.

4. Result and Discussion

In convection heat transfer, mixed convection is defined as a combination of forced convection and natural convection heat transfer. In thermal convection problems, the Richardson number ($Ri=Gr/Re^2$) is a dimensionless parameter that represents the importance of natural convection relative to forced convection. If the Richardson number is much less than unity, the buoyancy force term is unimportant and natural convection is negligible ($Ri<0.1$), for high Richardson numbers($Ri>10$), natural convection is dominant and forced convection is negligible, if Richardson number is of order unity ($0.1<Ri<10$), then forced and natural convection are important.

The presence of dense nanotube particles with high thermal conductivity in suspensions has different effects on temperature and flow fields. The effect of different volume fraction of added nanotubes on flow and temperature fields is presented in Figures.7, 8 and 9. Figure.7 shows the variation of temperate and velocity components at horizontal mid-section of the cavity for different Richardson numbers when the aspect ratio is equal to unity. It is visible that adding cylindrical nanoparticles to the base fluid has more obvious effects at high Richardson numbers that natural convection is dominant which is due to the role of dense particles on fluid flow when natural convection regime is dominant and buoyancy force term has more importance.

The streamline and temperature contours are presented in Figs.8 and 9, respectively, for pure fluid and nanofluid with about 0.75% volume fraction of single-walled Carbon nanotubes. At the present study to be sure that nanofluid behavior is completely same as single phase fluid, the volume fraction value of added nanotubes is less than 1% to make a dilute suspension, therefore it can be predicted that the overall shape of the contours are same for both pure fluid and nanofluid with added nanotube particlesThe values of average Nusselt number for different aspect ratio of the cavity are plotted in Fig.10 at different volume fractions and Richardson numbers. It is found that the presence of a low value of Carbon nanotubes enhances the heat transfer rate obviously. Adding the 0.5% volume fraction of SWCNT-particles enhances the average Nusselt number about 15%, 14% and 13% at $Ri=0.1$, $Ri=1$ and $Ri=10$, respectively, and these values are about 28%, 27% and 25% for the

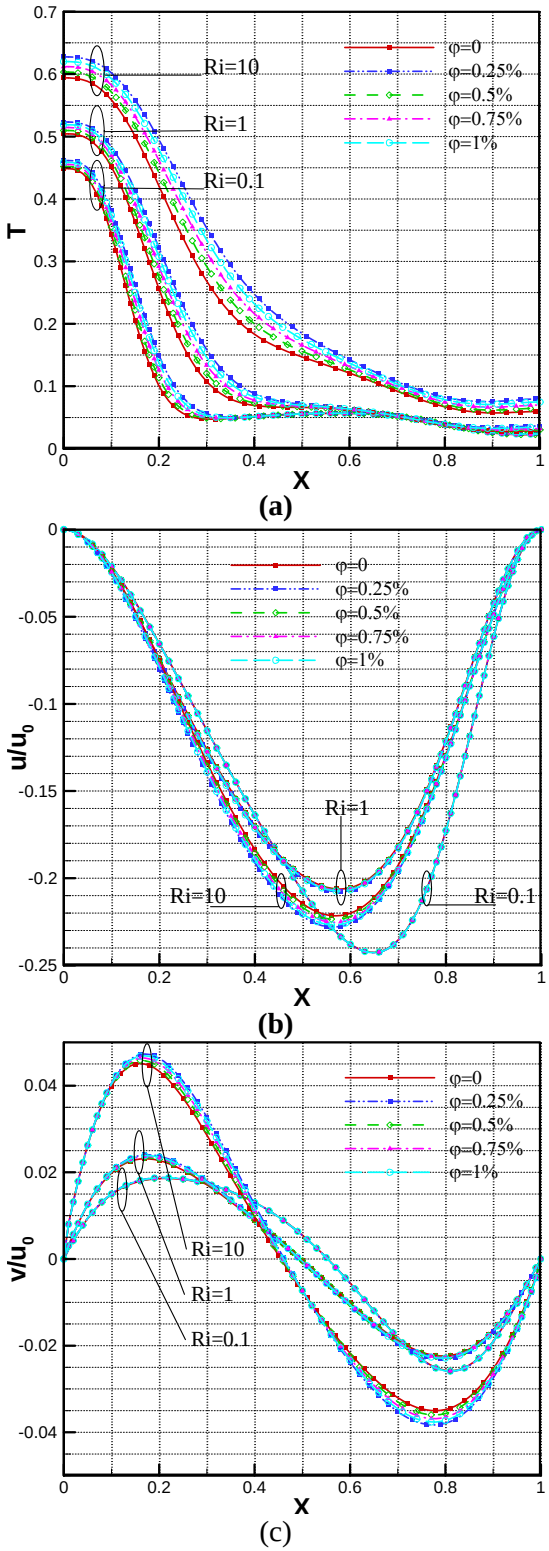


Fig.7. Effect of different volume fractions of nanotubes on Temperature (a), u-Velocity (b), v-Velocity (c) in horizontal mid-section of the cavity at different R_i and $A=1$.

for the volume fraction equal to 1% when the cavity aspect ratio is equal to unity. Improvement of thermal conductivity of the base fluid and increasing of flow momentum are some of the advantages of nanoparticles presence in the base fluid. These increases in flow momentum and thermal conductivity lead to heat transfer enhancement of the base fluid. As presented the volume fraction value of added single-walled Carbon nanotubes is less than 1 % that means significant effects on convection heat transfer and its flow are observed by adding a low value of added nanotubes which is due to extraordinary thermal conductivity and other properties of the Carbon nanotubes. Make an overall comparison between observed heat transfer enhancement due to adding Carbon nanotubes in this study and heat transfer enhancement of other nanoparticles achieved in published papers shows that Carbon nanotubes have great effects on heat transfer rate of conventional fluids as was predicted before.

The comparison of average Nusselt number value for different aspect ratio shows that by increasing of the aspect ratio, the rate of mixed convection heat transfer reduces for all studied volume fraction and Richardson numbers (Fig.10). Make a comparison between streamline contours presented in Fig.8 at different aspect ratio reveals that at , the flow in the cavity domain is mono-cellular for then by increasing of the aspect ratio to a small and week vortex at the bottom of cavity is created. Then increase of the aspect ratio led to this small vortex growths up. The creation of a week vortex at near of hot bottom wall and its growth up by increase of cavity's aspect ratio causes to heat transfer rate decrease. At other hand, for higher Richardson number the physical condition of streamlines is different from low Richardson numbers. It is visible (Fig.8) that for $Ri = 10$, although the flow pattern is mono-cellular at all aspect ratios, the strength of stream function decrease by an increase of the aspect ratio. According to this matter, increasing of cavity's aspect ratio can lead to heat transfer reduction in high Richardson numbers. Also, to expand the details of heat transfer reduction by increasing of cavity's aspect ratio, it must be mentioned that distance between hot and cold walls has an important role for conduction and convection heat transfer at the problem domain. An increase of distance between hot and cold walls can be cited as an important reason for heat transfer reduction by increasing of the aspect ratio at all Richardson numbers.

To delineate the details of heat transfer reduction by increasing of the cavity aspect ratio, the Nusselt number distribution along the cavity lid wall is presented in Fig.11 for different Richardson numbers. As can be seen in this figure, an increase in the aspect ratio led to Nusselt number reduction for all Richardson numbers. The variation rate of Nusselt number by changing aspect ratio at low Richardson numbers is higher than high Richardson numbers (Fig.11).

5. Conclusion

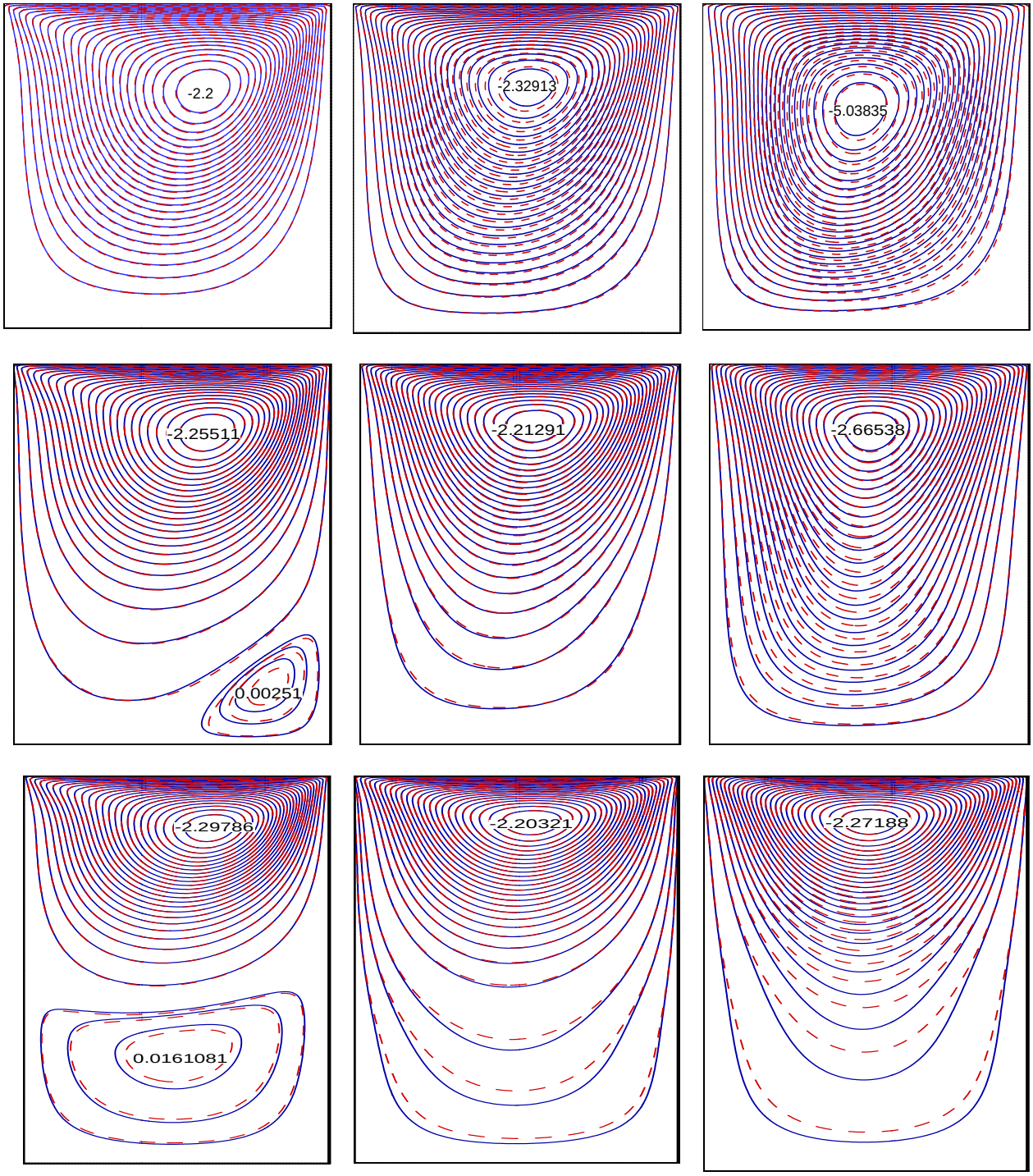
The effect of single walled Carbon nanotubes on mixed convection heat transfer in a lid-driven deep cavity was studied numerically. The double population approach in lattice Boltzmann Method was used at this study and the Bounce Back Method with second order accuracy in both flow and temperature field is applied to model solid boundaries. The problem was investigated at different Richardson numbers ($0.1 \le Ri \le 10$), aspect ratio of the cavity ($0.5 \le A \le 2.5$) and the volume fraction of SWCNT ($0 \le \varphi \le 1\%$) when Grashof number was fixed to 10^3. Some of most important results that have been achieved in this study are as follows:

(a) The Lattice Boltzmann Method is a suitable numerical method to model convection heat transfer of Carbon nanotube based nanofluids.

(b) Single walled Carbon nanotubes enhance the rate of convection heat transfer even with a low volume fraction.

(c) Increase of volume fraction of added SWCNT-particles led to Nusselt number increase.

(d) The aspect ratio of the cavity plays an important role on mixed convection heat transfer. An increase of this parameter leads to heat transfer reduction in a 2D deep lid-driven cavity.



(a) **(b)** **(c)**

Fig.8. Streamline contour at different Richardson numbers and aspect ratios.

a) $Ri = 0.1$ b) $Ri = 1$ c) $Ri = 10$ [Top) $A = 1$, Mid) $A = 1.5$, Bottom) $A = 2$]

($\varphi = 0$ $\varphi = 0.75\%$)

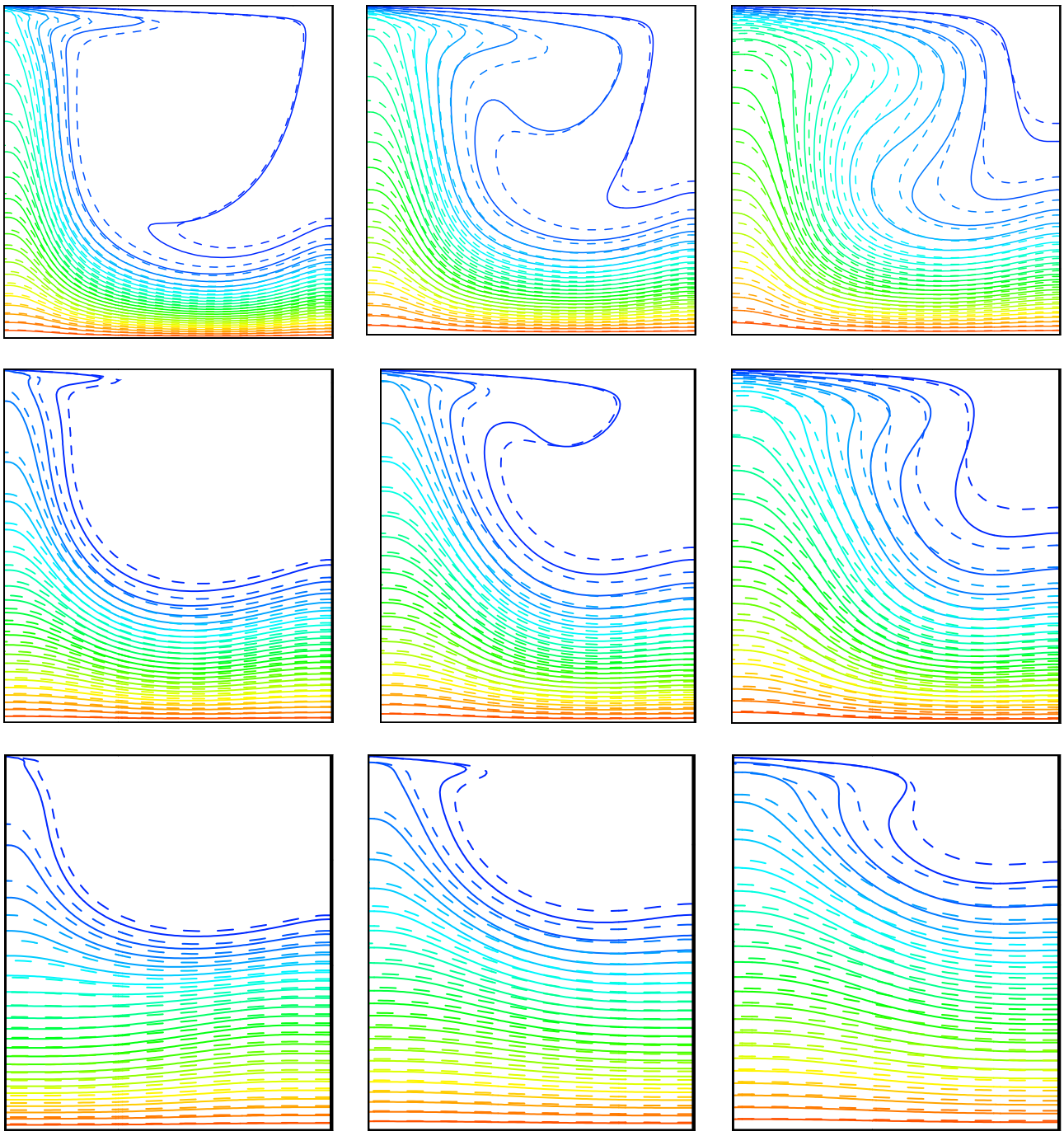


Fig.9. Temperature contour at different Richardson numbers and aspect ratios.
a) $Ri = 0.1$ b) $Ri = 1$ c) $Ri = 10$ [Top) $A = 1$, Mid) $A = 1.5$, Bottom) $A = 2$]
(——— $\varphi = 0$ - - - - - $\varphi = 0.75\%$)

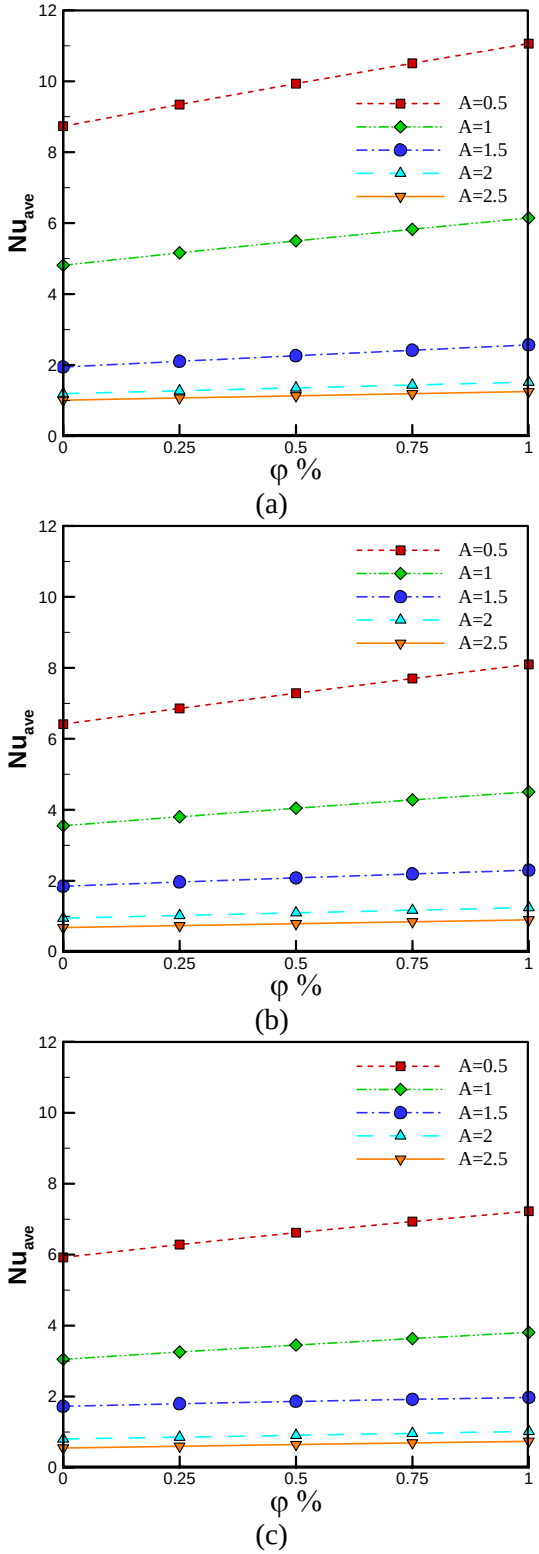


Fig.10. Average Nusselt number at different the aspect ratios and volume fractions of nanotubes.
a) $Ri = 0.1$ b) $Ri = 1$ c) $Ri = 10$.

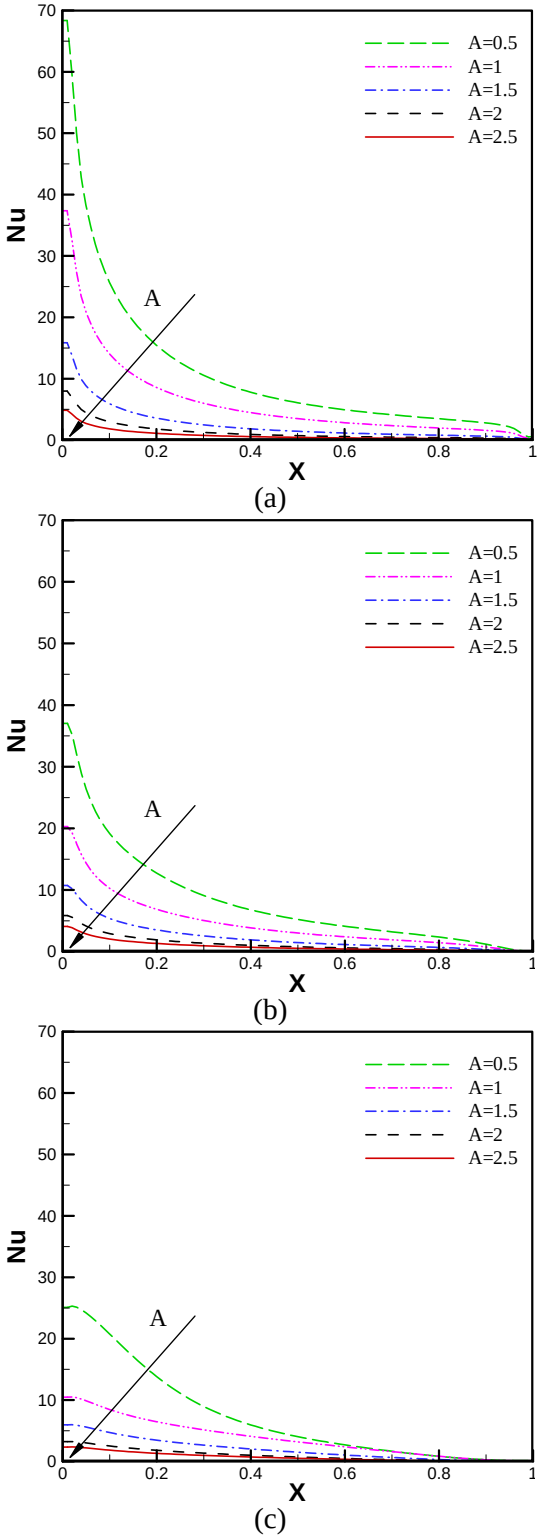


Fig.11. Local Nusselt number distribution along the cavity lid at different aspect ratios for nanofluid with $\varphi = 0.01$.
a) $Ri = 0.1$ b) $Ri = 1$ c) $Ri = 10$.

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