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ORIGINAL RESEARCH PAPER

Numerical Study of Natural Convection in a Square Cavity Filled with a Porous Medium Saturated with Nanofluid

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Abstract

Steady state natural convection of Al_2O_3-water nanofluid inside a square cavity filled with a porous medium is investigated numerically. The temperatures of the two side walls of the cavity are maintained at T_H and T_C, where T_C has been considered as the reference condition. The top and the bottom horizontal walls have been considered to be insulated i.e., non-conducting and impermeable to mass transfer. Darcy–Forchheimer model is used to simulate the momentum transfer in the porous medium. The transport equations are solved numerically with finite volume approach using SIMPLER algorithm. The numerical procedure is adopted in the present study yields consistent performance over a wide range of parameters (Rayleigh number, Ra, $10^4 \leq Ra \leq 10^6$, Darcy number, Da, $10^{-5} \leq Da \leq 10^{-3}$, and solid volume fraction, φ, $0.0 \leq \varphi \leq 0.1$). Numerical results are presented in terms of streamlines, isotherms and average Nusselt number. It was found that heat transfer increases with increasing of both Rayleigh number and Darcy number. It is further observed that the heat transfer in the cavity is improved with the increasing of solid volume fraction parameter of nanofluids.

Keywords: *Nanofluid; Natural Convection; Numerical Study; Porous Medium; Square Cavity*

1. Introduction

Heat and fluid flow in cavities filled with porous media are famous natural phenomenon and have concerned of many researchers due to its many practical situations. Among these insulation materials, geophysics applications, building heating and cooling operations, underground heat pump systems, solar engineering and material science can be listed. Pop and Ingham [1], Bejan et al. [2], Vafai [3,4], Vadasz [5], Varol et al. [6,7] and Basak et al. [8, 9]. In addition, Basak et al. [10] studied the natural convection flow in a square cavity filled with a porous medium numerically using penalty finite element method for uniformly and non-uniformly heated bottom wall, and adiabatic top wall maintaining constant temperature of cold vertical walls. They used Darcy–Forchheimer model to simulate the momentum transfer in the porous medium. They found that the heat transfer is primarily due to conduction for $Da \leq 10^{-5}$ irrespective of Ra and Pr. They conclude that for convection dominated regimes at high Rayleigh numbers, the correlations between average Nusselt number and Rayleigh numbers are power law. In recently year, Basak et al. [11] studied the mixed convection flows in a lid-driven square cavity filled with porous medium numerically using finite element. They analyzed the influence of convection with Peclet

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Nomenclature

Cp	Specific heat(J/kg K)
Da	Darcy number
g	Gravitational acceleration(m/s^2)
Gr	Grashof number
h	Heat transfer coefficient($W/m^2 K$)
H	Enclosure length, m
k	Thermal conductivity(W/m K)
Nu	Nusselt number
p	Pressure(N/m^2)
P	Dimensionless pressure
Pr	Prandtl number
Ra	Rayleigh number
Re	Reynolds number
T	Temperature(K)
u,v	Velocity components(m/s)
U,V	Dimensionless velocity components
x,y	Cartesian coordinates(m)
X,Y	dimensionless Cartesian coordinates

Greek Symbols

α	Thermal diffusivity(m2/s)
β	Thermal expansion coefficient(K-1)
μ	Dynamic viscosity(kg/m s)
ν	Kinematic viscosity(m2/s)
ψ	Volume fraction of nanoparticles
ρ	Density(kg/m3)
θ	Dimensionless temperature

Subscripts

avg	Average
c	Cold
eff	Effective
f	Fluid
h	Hot
nf	Nanofluid
s	Solid particle

number.

They investigated that effect of Peclet numbers have been further for both natural convection and forced convection dominant regimes at high *Da* and also strong coupling between flow fields and temperature are exist at high *Pe*. They concluded that at $Da = 10^{-3}$, local Nusselt numbers show almost uniform and low values for low Peclet numbers and localized enhanced heat transfer rates are observed for high Peclet numbers. On the other hand, a technique for improving heat transfer is using solid particles in the base fluids, which has been used recently. The term nanofluid, first introduced by Choi [12], refers to fluids in which nanoscale particles are suspended in the base fluid. He offered that introducing nanoparticles with higher thermal conductivity into the base fluid results in a higher thermal performance for the resultant nanofluid. It is expected that the presence of the nanoparticles in the nanofluid increases its thermal conductivity and therefore, substantially enhances the heat transfer characteristics of the nanofluid [12]. A large numbers of studies have been devoted to natural and mixed convection heat transfer of nanofluids inside enclosures [13, 14]. Khanafer et al. [15] are among the first investigators who studied the natural convective heat transfer inside rectangular cavities filled with nanofluids numerically. They showed that, for various Rayleigh numbers in laminar flow regime, the heat transfer increased by increasing the volume fraction of the nanoparticles.

Based on literature reviews, despite a large number of numerical studies on free convection of nanofluids inside cavities with different boundary conditions, there is no investigated of Darcy number, Rayleigh number and volume fraction of nanoparticles in square cavities filled with a isotropic porous medium that saturated with water-Al_2O_3 nanofluid. The purpose of the present paper is aimed at a better understanding of such flow in an isotropic porous medium. The results in the form of streamlines and isotherms plots, average Nusselt number are presented for a wide range of Rayleigh numbers, Darcy number and volume fraction of the nanoparticles.

2. Problem formulation

The steady state, laminar, natural convection fluid flow and heat transfer within a porous medium of square cavity filled with Al_2O_3-water nanofluid is simulated numerically using the finite volume method. Each side of the square cavity, which is

displayed in Fig.1, is denoted H. The length of the cavity perpendicular to the plane of the figure is assumed to be long; hence, the problem is considered to be two-dimensional. The left and the right vertical walls of the cavity are maintained at constant temperatures T_h and T_c (T_h>T_c), respectively. The cavity's top and bottom walls are insulated.

Table 1
Thermophysical properties of water and nanoparticles at T=25°c.

Physical property	Water	Nanoparticles (Al_2O_3)
c_P (J/kg)	4179	765
ρ (kg/m^3)	997.1	3970
k (W/m K)	0.613	40
β (K^{-1})	21×10^{-5}	0.85×10^{-5}
μ(kg/m s)	8.55×10^{-4}	---

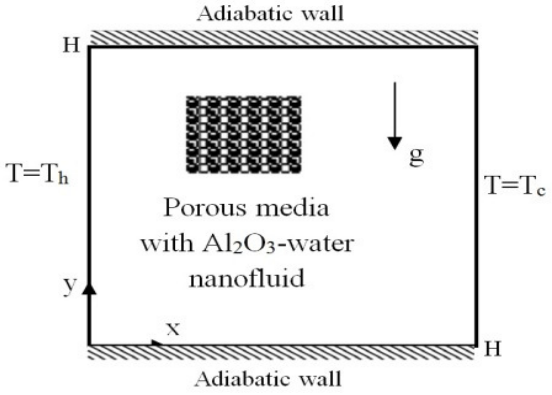


Fig. 1. Sketched of the physical model

The porous medium of cavity is filled with a nanofluid composed of a mixture of water and Al_2O_3 spherical nanoparticles. Darcy–Forchheimer model is used to simulate the momentum transfer in the porous medium. The nanofluid is assumed to be incompressible and Newtonian. The nanoparticles are presumed to be in thermal equilibrium with the base fluid. Moreover, there is no slip between the nanoparticles and the base fluid. Thermophysical properties of the base fluid and the nanoparticles are presented in Table 1.

3. Mathematical modeling

The density of the nanofluid is assumed to vary according to the Boussinesq approximation [16].

In order to cast the governing equations into a dimensionless form, the following dimensionless variables are introduced:

$$X = \frac{x}{H},\ Y = \frac{y}{H},\ U = \frac{uH}{\alpha_f},\quad V = \frac{vH}{\alpha_f},\ P = \frac{pH^2}{\rho_{nf}\alpha_f^2},\ \theta = \frac{T-T_c}{T_H-T_c} \tag{1}$$

Employing the dimensionless variables, the continuity, momentum, and energy equations for the nanofluid, incorporating the natural convection through the Boussinesq approximation in the y-momentum equation, become

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{2}$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\frac{\mu_{nf}}{\rho_{nf}\alpha_f}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right)-\frac{\mu_{nf}}{\rho_{nf}\alpha_f Da}U \tag{3}$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\frac{\mu_{nf}}{\rho_{nf}\alpha_f}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)-\frac{\mu_{nf}}{\rho_{nf}\alpha_f Da}V+Ra\,\Pr\frac{\rho_f}{\rho_{nf}}(1-\varphi+\varphi\frac{\rho_s\beta_s}{\rho_f\beta_f})\theta \tag{4}$$

and

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{\alpha_{nf}}{\alpha_f}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \tag{5}$$

where ρ_f, μ_f, β_f, and α_f are the density, the viscosity, the thermal expansion coefficient, and the thermal diffusivity of the base fluid, respectively. The Rayleigh number, Darcy number and Prandtl number are defined, respectively, as

$$Ra=\frac{g\beta_f H\Delta T}{\upsilon_f\alpha_f},\ Da=\frac{k}{H^2},\quad and\ \ Pr=\frac{\upsilon_f}{\alpha_f}. \tag{6}$$

The Prandtl number of water is Pr = 5.83.

The boundary conditions for Eqs. (2)–(5) are

$$Y=0,1\ ,\ 0\le X\le 1 \quad \rightarrow U=V=\frac{\partial\theta}{\partial Y}=0 \tag{7a}$$

$$X=0,\ 0\le Y\le 1 \rightarrow V=U=0,\ \theta=1 \tag{7b}$$

$$X=1,\ 0\le Y\le 1 \ \rightarrow V=U=\theta=0 \tag{7c}$$

The density, ρ_{nf}, the heat capacity, $(\rho c_p)_{nf}$, and the thermal expansion coefficient, $(\rho\beta)_{nf}$, of the nanofluid are obtained from the following equations [15]:

$$\rho_{nf}=(1-\varphi)\rho_f+\varphi\,\rho_s \tag{8}$$

$$(\rho c_p)_{nf}=(1-\varphi)(\rho c_p)_f+\varphi(\rho c_p)_s \tag{9}$$

$$(\rho\beta)_{nf}=(1-\varphi)(\rho\beta)_f+\varphi(\rho\beta)_s \tag{10}$$

To estimating the dynamic viscosity of the nanofluid the Brinkman model [17] is employed.

$$\mu_{nf}=\frac{\mu_f}{(1-\varphi)^{2.5}} \tag{11}$$

The effective thermal conductivity of the nanofluid (k_{eff}) has been determined by the model proposed by Patel et al. [18], as the following relation:

$$\frac{k_{eff}}{k_f}=1+\frac{k_p A_p}{k_f A_f}+ck_p Pe\frac{A_p}{k_f A_f} \tag{12}$$

$$\frac{A_p}{A_f}=\frac{d_f}{d_p}\frac{\varphi}{(1-\varphi)}\ ,\ Pe=\frac{u_p d_p}{\alpha_f}$$
$$and \quad u_p=\frac{2k_bT}{\pi\mu_f d_p^2} \tag{13}$$

In the above relations, d_p and d_f, which are equal to 47 nm and 0.384 nm, respectively, are the diameters of the Al_2O_3 nanoparticles and water molecule, respectively. u_p is the speed of the Brownian motion of the nanoparticles, K_b is the Boltzmann constant which is equal to 1.38×10^{-23}, and c is an empirical constant which is taken to be 2.5×10^4 [19]. The thermal diffusivity of the nanofluid is expressed as

$$\alpha_{nf}=k_{nf}/\left(\rho c_p\right)_{nf} \tag{14}$$

The local and average Nusselt number is obtained from the following relations where N is the direction of normal to the wall.

$$Nu=-\frac{k_{eff}}{k_f}\frac{\partial\theta}{\partial N}\bigg|_{wall} \tag{15}$$

$$Nu_{avg}=\int_0^1 Nu_y\,dY\ \Big|_{X=0} \tag{16}$$

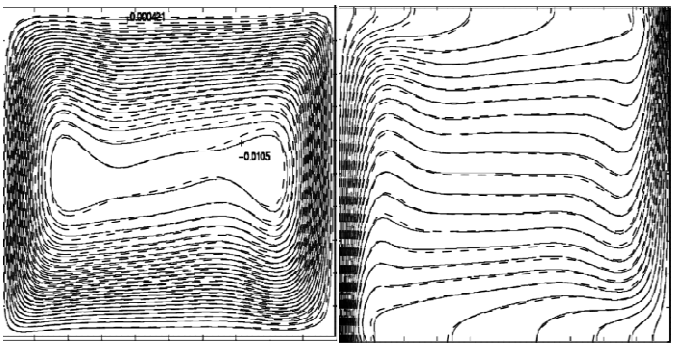
4.Numerical scheme

The governing equations are discretized using the finite volume method. Coupling between the pressure and the velocity is done using the SIMPLER algorithm. The diffusion terms in the equations are discretized by a second order central difference scheme, while a hybrid scheme (a combination of the central difference scheme and the upwind scheme) is employed to approximate the convection terms. The set of discretized equations are solved by TDMA and line by line method [20]. In order to determine a suitable grid for the numerical simulation, a porous medium in a square cavity filled with Al_2O_3-water nanofluid with $Da=10^{-3}$ and $\varphi=0.1$ at $Ra = 10^6$ is chosen. Five different uniform grids, namely, 21×21, 41×41, 61×61, 81×81, and 101×101 are employed for the numerical simulations. The average Nusselt numbers of hot left wall for these grids are shown in Table 2. As can be observed from the table, a uniform 81×81 grid is sufficiently fine for the numerical calculation.

Table 2
Average Nusselt number of the hot wall for different

Number of nodes	21×21	41×41
Nu_{avg}	7.152	7.709

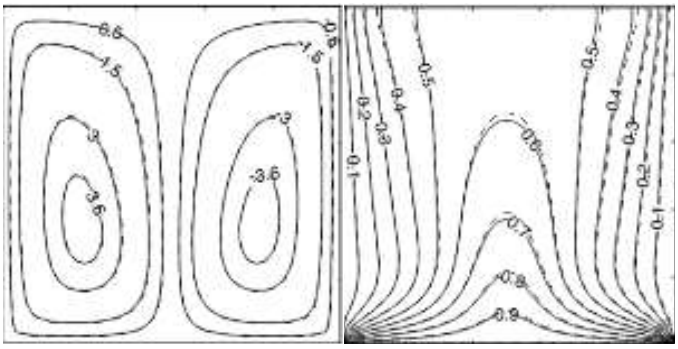
In order to validate the numerical procedure, two test cases are considered. In the developed code the first test case is free convection in square cavity filled with Cu-water nanofluid with cold right wall, heated left wall and insulated horizontal walls. The obtained results using the presented code are compared with the results of Khanafer et al. [15] for the same problem. Comparisons between the streamlines and the isotherms inside the cavity obtained by the present code and the results of Khanafer et al [15] for $\varphi=0.08$ and $Gr=10^5$ are presented in Fig.2. As can be observed from the figure, very good agreements exist between the two results.



(a) (b)

Fig. 2. (a) Streamlines and (b) isotherms for $Gr=10^5$ and $\varphi=0.08$: comparison present results (---) with the result of Khanafer et al. [15] (—).

In another developed code, the second test case is Natural convection flows in a square cavity filled with an isotropic porous medium. It is assumed that the bottom wall is heated uniformly and non-uniformly while the top wall is well insulated and the vertical walls are cooled to a constant temperature. The obtained results using the presented code are compared with the results of Basak et al. [10] for the same problem. Fig.3. shows the Comparisons between the streamlines and the isotherms inside the cavity obtained by the present code and the results of Basak et al [10] for $\theta(X,0)=1$, $Ra=10^6$, $Pr=0.71$ and $Da=10^{-4}$. As the figure shows, very good agreements exist between the results in this case two.



(a) (b)

Fig. 3. (a) Streamlines and (b) isotherms for $\theta(X,0)=1$, $Ra=10^6$, $Pr=0.71$ and $Da=10^{-4}$: comparison present results (---) with the result of Basak et al. [10] (—).

5. Results and discussion

In this section, numerical simulation results on flow field, temperature distribution, and average Nusselt number are presented. The study focuses on effects of the Rayleigh number, Darcy number, and volume fraction of the nanoparticles on the flow and temperature fields. The Rayleigh number, the Darcy number, and the nanoparticles volume fraction are ranging from 10^4 to 10^6, 10^{-5} to 10^{-3} and 0.0 (pure fluid) to 0.1, respectively.

The streamlines and isotherms inside the cavity for a range of Ra from 10^4 to 10^6, Darcy number from 10^{-5} to 10^{-3}, and for pure fluid and nanofluid namely $\varphi=0.0$ and $\varphi=0.1$ are displayed in Fig. 4.

In general, the presence of nanoparticles in the fluid is found to alter the structure of the fluid flow. In other words, adding nanoparticels to pure fluid or porous medium can impress heat transfer and fluid flow but with different behaviors for each medium. It is observed that a single circulation flow cell is formed in the clockwise direction for all values of Ra that has been tested. In general, the pure fluid and also nanofluid circulation is strongly dependent on Darcy number as we have seen in Fig. 4. For all range of Ra especially at high Ra, the flow is seen to be very weak at low Darcy number (10^{-5}) as observed from stream function contours. As Darcy number increases to 10^{-4} and 10^{-3}, the strength of flow is increased. At low Darcy number the temperature distribution is similar to that with stationary fluid and the heat transfer is due to purely conduction. As Darcy number increases to 10^{-4} and 10^{-3}, the stronger circulation causes the temperature contours to be concentrated near the side walls which may result in greater heat transfer rate due to convection and it is more specified at $Ra = 10^6$.Therefore, at $Da=10^{-3}$, the convection dominant heat transfer mode would be occur.

Rayleigh number is a very important parameter that has effects on heat transfer of nanofluid within a porous medium. When the Rayleigh number is low, the flow convection is insignificant. The heat transfer in the cavity is dominated by conduction. At $Ra = 10^4$ and for all values of Da, via domination of conduction heat transfer the isotherms are nearly parallel with the vertical walls, for the cavity filled with pure fluid and nanofluid. By increase of the buoyant force via increase in the Rayleigh number, the flow intensity increases and the streamlines closes to the side walls. At $Ra = 10^6$ the streamlines are located close to the isothermal side walls and distinct velocity boundary layers in this region. The isotherms in Fig. 4 indicate that with increase in the Rayleigh number, the effect of free convection increases and the isotherms are condensed next to the

isothermal side walls. Moreover, thermal stratification is observed in the left and right sides of the square body. Formation of the thermal boundary layers can be observed from the isotherms at Ra = 10^6.

At Ra=10^4 with Da=10^{-3}, by increasing the volume fraction of the nanoparticles we have seen difference in isotherms plots of pure fluid with nanofluid, because of the high conduction of nanoparticles. There have been differing at Da=10^{-3} due to less resistances of porous medium against the fluid flow, while at Da=10^{-4}, 10^{-5} difference in isotherms of pure fluid and nanofluid does not exist.

From the streamlines at every Rayleigh number it can be observed that the velocity boundary layers adjacent to the isothermal walls are thicken when the nanoparticles volume fraction increases. It is because of increase in diffusion of mass via increase in viscosity of nanofluid. As well as, with increasing the volume fractions of nanoparticles the vortex strength at the core of the cavity has been decreased. On the other hand, at Ra=10^6, the vortex at core of the cavity is get smaller while becoming weaker as the volume fraction of the nanoparticles increases and, consequently, the inertia and viscous resistances of the nanofluid increase. At Ra=10^6 with increasing the Darcy number from 10^{-5} to 10^{-3}, the temperature gradients near the side walls tend to be significant to develop the thermal boundary layer. It can be seen from the isotherms that at each Rayleigh number, with increase in the nanoparticles volume fraction the thermal boundary layers thicken via increase in the thermal conductivity of the nanofluid, reduction of the temperature gradient adjacent to the side walls and increase in diffusion of heat.

Variations of the average Nusselt number on the hot wall of the cavity with the volume fraction of the nanoparticles for different Rayleigh numbers and Darcy numbers are shown in Fig. 5.

The average Nusselt number is almost elevated when the Rayleigh number increases and nanoparticles are used. However, the effects of solid volume fraction parameter φ on the heat transfer of nanofluids are complicated. When Rayleigh number Ra is low (Ra = 10^4), at all range of Darcy number Da the average Nusselt number increases as solid volume fraction (φ) increases. At these Rayleigh number, the rate of heat transfer increases with increase in the volume fraction of the nanoparticles. At Ra=10^5 and Da=10^{-5}, 10^{-4} the average Nusselt number increases as solid volume fraction increases. But at these Ra and Da=10^{-3} the average Nusselt number is constant when the volume fraction of the nanoparticles increases. At Ra=10^6 and Da=10^{-5}, 10^{-3} the average Nusselt number increases with increasing in volume fraction of nanoparticles, although at Da=10^{-4} the average Nusselt number decreases with increasing in volume fraction of nanoparticles.

Conclusions

In the present paper the problem of natural convection of Al_2O_3-water nanofluid inside differentially heated square cavity filled with porous medium was investigated numerically using the finite volume method and SIMPLER algorithm. The momentum transfer in the porous region is modeled by using Darcy–Forchheimer law. A parametric study was undertaken and effects of the Rayleigh number, the Darcy number, and the volume fraction of the Al_2O_3 nanoparticles on the fluid flow, temperature field and rate of heat transfer were investigated and the following results were obtained.

- For all considered cases, when the volume fraction of the nanoparticles is kept constant, the rate of heat transfer increases by increase of the Rayleigh number.
- At Ra=10^4, 10^5, and 10^6, when the both Rayleigh number and volume fraction of the nanoparticles are kept constant, the average Nusselt number increases by increasing the Darcy number from 10^{-5} to 10^{-3}.
- At Ra=10^4 and all values of Darcy number, the heat transfer improved by increasing the volume fraction of the nanoparticles.
- At Ra=10^5, the average Nusselt number increases by increasing the volume fraction of the nanoparticles except for Da=10^{-3} that keep constant.
- At Ra=10^6, the average Nusselt number increases by increasing the volume fraction of the nanoparticles except for Da=10^{-4}.
- At Da=10^{-3}, due to low resistances of porous medium against the fluid flow, the strength of convection increased accordingly, the average Nusselt number increases but the effect is more efficacious by nanofluid than pure fluid

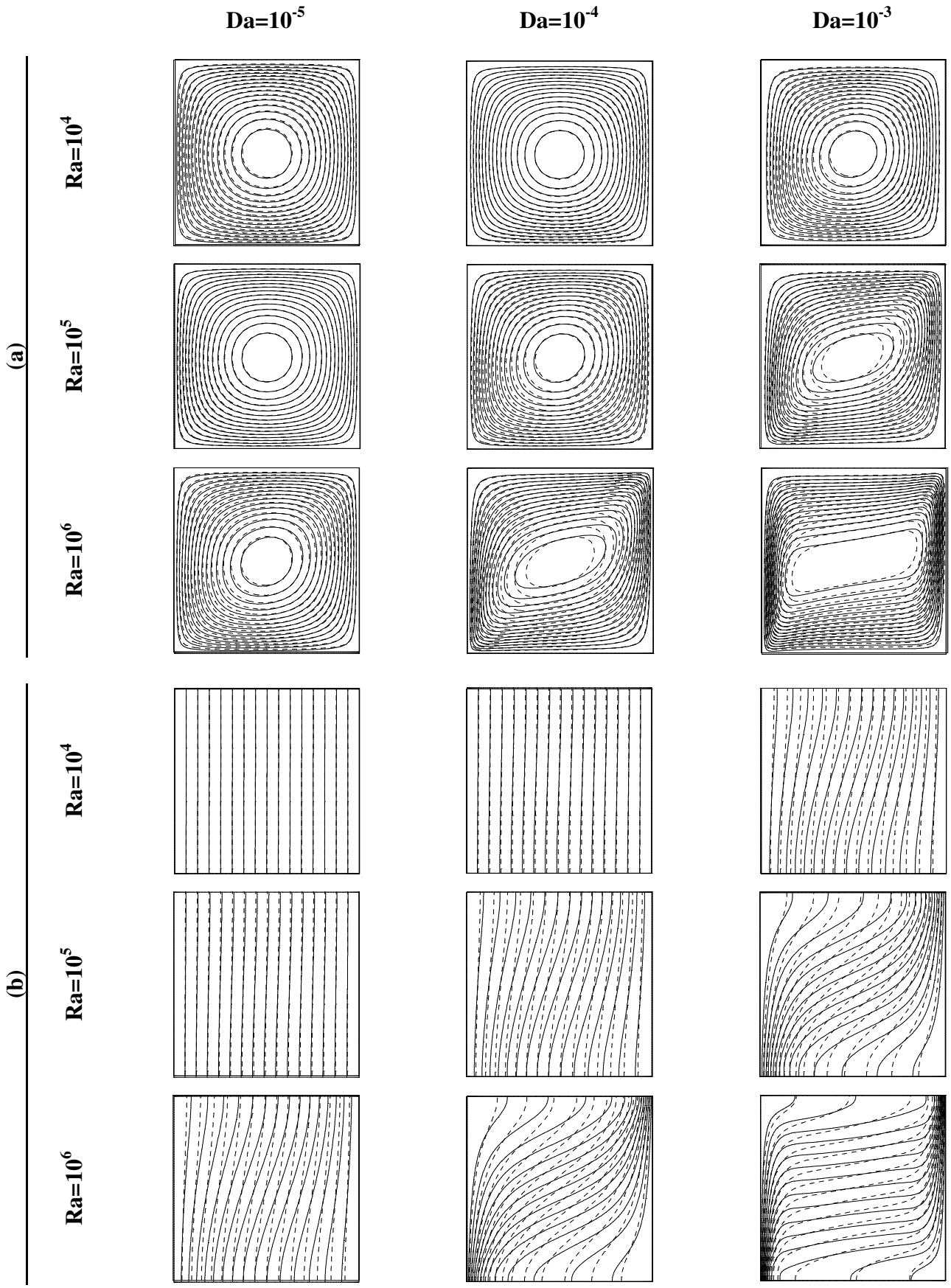


Fig. 4. (a)Streamlines and (b) isotherms inside the cavity for φ=0.0 (—), and 0.1 (---), at different Rayleigh, and Darcy numbers.

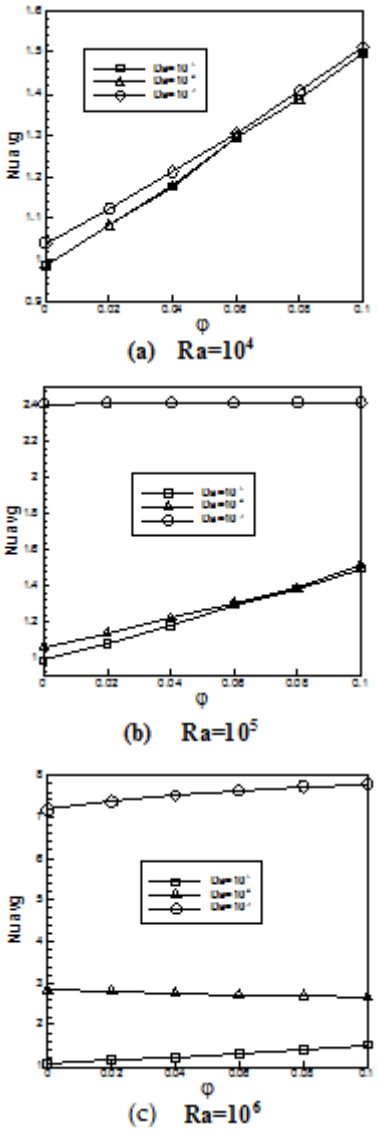


Fig. 5(a-c). Variation of the average Nusselt number with the volume fraction of the nanoparticles at different Rayleigh numbers and different Darcy numbers.

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